**Induction Process:**

1. Prove the statement is true for $n = 1$
   (this is your base case, and can sometimes start with another value, say $n_0$).
2. Assume the statement is true for $k$
3. Prove the statement is true for $k + 1$

**Example:** For each $n$ in the Natural numbers, $2^n \geq n + 1$.

1. For $n = 1$, prove $2^n \geq n + 1$
   
   $2 = 2^1 \geq 1 + 1 = 2$
2. Assume $2^k \geq k + 1$ for any natural number $k$
3. Prove $2^{k+1} \geq (k+1) + 1$
   
   $2^{k+1} = 2^k \cdot 2 \geq (k + 1) \cdot 2 = 2k + 2$
   
   Since $k$ is a natural number, $k > 0 \Rightarrow 2k - k > 0 \Rightarrow 2k > k$
   
   Therefore, $2^{k+1} \geq 2k + 2 > k + 2 = (k + 1) + 1$
   
   We have shown that $2^{k+1} > (k + 1) + 1 \Rightarrow 2^{k+1} \geq (k + 1) + 1$

Why not use a strict inequality?

**Example:** For each $n$ in the natural numbers, $\sum_{k=1}^{n} k^2 = \frac{1}{6} n(n + 1)(2n + 1)$

1. For $n = 1$ we want to prove that $\sum_{k=1}^{n} k^2 = \frac{1}{6} n(n + 1)(2n + 1)$
   
   $\sum_{k=1}^{1} k^2 = \frac{1}{6} \cdot 1(1+1)(2+1)$
   
   $1^2 = \frac{6}{6}$
2. Assume $\sum_{k=1}^{n} k^2 = \frac{1}{6} n(n + 1)(2n + 1)$
3. Prove $\sum_{k=1}^{n+1} k^2 = \frac{1}{6} (n+1)(n + 2)(2n + 3)$
   
   $\sum_{k=1}^{n+1} k^2 = \sum_{k=1}^{n} k^2 + (n + 1)^2 = \frac{1}{6} n(n + 1)(2n + 1) + (n + 1)^2$
   
   $= (n + 1) \left[ \frac{1}{6} n(2n+1) + (n + 1) \right] = (n + 1) \left[ \frac{2n^2 + n + 6n + 6}{6} + \frac{6}{6} \right]$
   
   $= \frac{1}{6} (n+1)(2n^2 + 7n + 6) = \frac{1}{6} (n+1)(n+2)(2n+3)$