Section 6.3 – Elementary Row Operations and Determinants

Homework (pages 463-464) problems 1-19

Rules:

- **Theorem 1.** For an \( nxn \) matrix \( A \), \( \text{det}(A) = \text{det}(A^T) \)

- **Theorem 2.** If you interchange two rows (or columns) of an \( nxn \) matrix \( A \), it changes the determinant by a factor of \(-1\).

  - Example. If \( A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \) and \( B = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \) find \( \text{det}(A) \) and \( \text{det}(B) \)

    Solution: \( \text{det}(A) = 4 - 6 = -2 \). \( \text{det}(B) = 6 - 4 = 2 \)

- **Theorem 3.** If you multiply a row (or column) by a scalar \( c \), it changes the determinant by a factor of \( c \).

  - Example. If \( A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \) and \( B = \begin{bmatrix} 2 & 6 \\ 2 & 4 \end{bmatrix} \) find \( \text{det}(A) \) and \( \text{det}(B) \)

    Solution: \( \text{det}(A) = -2 \). \( \text{det}(B) = 8 - 12 = -4 = 2(-2) \)

  - This would imply that if you multiply an \( nxn \) matrix by a scalar \( c \), then \( \text{det}(cA) = c^n \text{det}(A) \)

- **Theorem 4.** If you add the \( s \)th row (or column) of matrices \( B \) and \( C \) (who are equivalent except for the row \( s \)) to yield a new matrix \( A \), then \( \text{det}(A) = \text{det}(B) + \text{det}(C) \).

  - Example. If \( B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \) and \( C = \begin{bmatrix} 3 & 1 \\ 3 & 4 \end{bmatrix} \) and \( A = \begin{bmatrix} 4 & 3 \\ 3 & 4 \end{bmatrix} \) find the determinants

    Solution: \( \text{det}(B) = -2 \), \( \text{det}(C) = 9 \). \( \text{det}(A) = 7 = \text{det}(B) + \text{det}(C) \)

- **Theorem 5.** If a row (or column) is a multiple of another row (or column) then the determinant is 0.

  - Example. If \( A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \) find \( \text{det}(A) \)

    Solution: \( \text{det}(A) = 0 \)

- **Theorem 6.** The determinant is not changed if a multiple of one row (or column) is added to another.

  - Example. If \( B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \) and \( A = \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix} \) and \( C = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix} \) find the determinants

    Solution: \( \text{det}(B) = -2 \) \( \text{det}(A) = -2 \) but \( \text{det}(C) = 2 \)… why?