**Section 2.4 – Lines and Planes in Space**

**Homework (pages 157-158) problems 1-24**

**Equation of a Line in Three Space:**

- In two space, we need a point and the slope of a line to find its equation.
- In three space, we need a point and the direction vector to define an equation.
- A point \( P = (x,y,z) \) is on the line [defined with point \( P_0 = (x_0,y_0,z_0) \), direction vector \( \vec{u} = (u_1,u_2,u_3) \) ] iff \( P_0P = t\vec{u} \) for some \( t \).

The vector \( P_0P = \begin{bmatrix} x-x_0 \\ y-y_0 \\ z-z_0 \end{bmatrix} = t \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \). Therefore, the **parametric equations of the line** are given by

\[
\begin{align*}
x &= x_0 + tu_1 \\
y &= y_0 + tu_2 \\
z &= z_0 + tu_3
\end{align*}
\]

This, in fact, defines any point on the line for particular values of \( t \).

- **Exercise 2:** Find the parametric equations for the line through \( P_0 = (1,1,-1) \) in the direction \( \vec{u} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} \)
- **Exercise 4:** Give parametric equations for the line through \( P_0 = (5,1,-3) \) and \( P_1 = (6,6,4) \)
- **Exercise 6:** Are the planes given by \( x = 2 + 6t \) and \( y = 7 + 2t \) parallel?

- Two lines are **parallel** if their direction vectors are scalar multiples of each other.

- **Exercise 6:** Are the planes given by \( y = 2 + 4t \) and \( y = 7 + 2t \) parallel?

- **Exercise 6:** Are the planes given by \( z = -1 + 3t \) and \( z = 2 + 2t \) parallel?

- To specify a **line segment** we restrict the values of \( t \).
**Planes in Space:**

- A vector is said to be **normal** to a plane if it is perpendicular to every vector in the plane.
- A plane is determined by a point and a normal vector.
- Also, a plane is uniquely defined by any three distinct points $P_0, P_1$ and $P_2$. We can create vectors in this plane by $\vec{u} = \overrightarrow{P_0P_1}$ and $\vec{v} = \overrightarrow{P_0P_2}$. A normal to the plane is given by $\vec{n} = \vec{u} \times \vec{v}$.
- For a plane with point $P_0 = (x_0, y_0, z_0)$, and normal $\vec{n}$, a point $P$ is in the plane iff $\vec{n} \cdot \overrightarrow{PP_0} = 0$.

  - For a plane with point $P_0 = (x_0, y_0, z_0)$, and normal $\vec{n} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$, a point $P = (x, y, z)$ is in the plane iff $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$.

- Two planes are **parallel** if their normals are parallel.
- **Exercise 10:** Find parametric equations for the line through $P_0 = (2,0,-3)$ and perpendicular to the plane given by $x - y + 2z = 8$.  
  \[ x = 4 + 2t \]

- **Exercise 12:** Find a point where the line $y = 7 + 2t$ intersects the given plane $x + y - z = 2$. 
  \[ z = 5 + 4t \]

- **Exercise 16:** Find the equation of the plane through $P_0 = (4,-2,3)$ with normal $\vec{n} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$.

- **Exercise 18:** Find the equation of the plane through $P = (5,1,7)$, $Q = (6,9,2)$ and $R = (7,2,9)$.

- **Exercise 22:** Find a unit normal for the plane $4x - 4y + 2z = 10$.

- **Exercise 24:** Find the equation of the plane through $P_0 = (1,1,3)$ and parallel to $2x - 3y + 2z = 6$.