Section 2.2 – Vectors in Space
Homework (pages 134-135) problems 1-35

The Basics:
- For any point P in space, we can locate it by its position on the Cartesian coordinate system with the ordered triple (x, y, z).
- Exercise 2: Plot the point P = (1,1,0).
- It is standard for the xy plane to be on the “bottom” so to speak, and the z axis pointing directly up.
- The right handed system is for x to be in “front” and y to run along the back.

- The vector \( \vec{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \) now denotes a vector in three space, where \((u_1, u_2, u_3)\) indicates the terminal point.
- The vector \( \vec{v} = \overrightarrow{AB} \) can be found with \( \begin{bmatrix} b_1 - a_1 \\ b_2 - a_2 \\ b_3 - a_3 \end{bmatrix} \).
- Exercise 6: Find the vector form for \( \overrightarrow{AB} \), where \( A = (1,0,3) \) and \( B = (3,2,5) \).
- Again, we have the x, y and z components of the vector (in above 3x1 matrix), respectively.
- The distance between two points \((x_1, y_1, z_1)\) and \((x_2, y_2, z_2)\) in space is given by
  \[ d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \].
- Exercise 2: Find the distance between (1,1,0) and (0,0,1).
- We can find the midpoint of two points in three space with \( M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right) \).
- Exercise 6: For the vector \( \overrightarrow{AB} \), where \( A = (1,0,3) \) and \( B = (3,2,5) \), calculate the distance of the midpoint to the origin.
Operations Carry Over:  

- To add geometric vectors \( \vec{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \) and \( \vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \) we simply add the components, \( \vec{u} + \vec{v} = \begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \\ u_3 + v_3 \end{bmatrix} \).

- To multiply a vector by a scalar, \( c\vec{u} = \begin{bmatrix} cu_1 \\ cu_2 \\ cu_3 \end{bmatrix} \).

- To find a unit vector with the same direction as \( \vec{u} \), we can find \( \frac{\vec{u}}{\|\vec{u}\|} \).

Basis Vectors  

- The unit vectors given in the direction of the positive \( x \), \( y \) and \( z \) axes are the \textbf{basic vectors}.

- They are given by \( \vec{i} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \), \( \vec{j} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \) and \( \vec{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \).

- Any vector \( \vec{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \) can be expressed in terms of the unit vectors by \( \vec{u} = u_1\vec{i} + u_2\vec{j} + u_3\vec{k} \).

Some Example Problems:  

- For \( \vec{u} + 2\vec{v} \), \( \|\vec{u} - \vec{v}\| \), \( \vec{w} \) so that \( \vec{u} + 2\vec{w} = \vec{v} \) for \( \vec{u} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \), \( \vec{v} = \begin{bmatrix} 4 \\ 3 \\ 6 \end{bmatrix} \).

- Find the vector who has the same direction as \( \vec{v} = \vec{i} + \vec{j} \) and magnitude \( \|\vec{u}\| = \sqrt{8} \).