Section 1.8 – Data Fitting
Homework (page 90) problems 1-6

Polynomial Interpolation:
- Recall a polynomial of degree \( n \) is of the form \( p(t) = a_0 + a_1 t + a_2 t^2 + \ldots + a_n t^n \).
- When given a table of values we can find an interpolation polynomial, which goes through all points given, and is of maximum degree one less than the number of points given.
- Example: Given the three points in the following table, find the interpolating polynomial:

<table>
<thead>
<tr>
<th>( t )</th>
<th>( f(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
</tr>
</tbody>
</table>

Solution:
Three points indicates this is a polynomial of degree 2, so it is of the form \( p(t) = a_0 + a_1 t + a_2 t^2 \). Plugging the above points \((t, f(t))\) into the equation for \( p(t) \) we find:
\[
a_0 + a_1 \cdot 1 + a_2 \cdot 1^2 = 5
\]
\[
a_0 + a_1 \cdot 3 + a_2 \cdot 3^2 = 11
\]
\[
a_0 + a_1 \cdot 4 + a_2 \cdot 4^2 = 14
\]
Putting this in matrix form, we have:
\[
\begin{bmatrix}
1 & 1 & 1 & 5 \\
1 & 3 & 9 & 11 \\
1 & 4 & 16 & 14 \\
\end{bmatrix}
\]
Row reducing, we find:
\[
\begin{bmatrix}
1 & 0 & 0 & 2 \\
0 & 1 & 0 & 3 \\
0 & 0 & 1 & 0 \\
\end{bmatrix}
\]
Therefore, our polynomial is given by \( p(t) = 2 + 3t \)

Does this check? How?

- Page 90, Exercise 6

NOTE: The matrix for up to 10 points can be checked with an Excel worksheet that is online.