Section 1.1 – Introduction to Matrices and Systems of Linear Equations
Homework (pages 12-13) problems 1-10, 30-36

Preliminary Problems:
• Solve the following set of equations: \(2x + 7y = 9\), and \(5x - 4y = 1\)
• How did you proceed? Was it easy to solve this, why or why not?
• How about the system given by:
  \[
  \begin{align*}
  4x - 2y + 8z &= 10 \\
  3x + 5y - 2z &= 6 \\
  12x + 12z &= 24
  \end{align*}
  \]
• Was this harder? Why or why not?

Linear Systems
• The above are examples of linear systems.
• A linear equation is a combination of terms \(a_ix_i\) where the \(x_i\) are the variables of the equation, and the terms are of degree one, and can only be added and subtracted. It is of the general form
  \[
  a_1x_1 + a_2x_2 + \ldots + a_nx_n = \sum a_ix_i = b.
  \]
• Page 12, Exercises 2 and 6
• A system of linear equations can be shorthand written in matrix notation, with each column identifying the variable, and each row identifying the equation number.
• For example, the above system given by
  \[
  \begin{align*}
  4x - 2y + 8z &= 10 \\
  3x + 5y - 2z &= 6 \\
  12x + 12z &= 24
  \end{align*}
  \]
can be represented by the augmented matrix
  \[
  \begin{bmatrix}
  4 & -2 & 8 & 10 \\
  3 & 5 & -2 & 6 \\
  12 & 0 & 12 & 24
  \end{bmatrix}
  \]
• We would identify this as a 3x4 matrix, where there are 3 rows and 4 columns.
• In general, a \(m \times (n+1)\) augmented matrix is written as
  \[
  \begin{bmatrix}
  a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\
  a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\
  \vdots & \vdots & \ddots & \vdots & \vdots \\
  a_{m1} & a_{m2} & \cdots & a_{mn} & b_n
  \end{bmatrix}
  \]
  It is called “augmented” because it contains the constant values to the right of the equal sign.
**Solutions of Systems**

- A system can have multiple, one, or no solutions.
- Recall from the “Preliminary Problem” that the solution to the 3x4 matrix is given by $(x, y, z) = (1, 1, 1)$.
- How can we verify this?
- A solution is a set of numbers that satisfies the system when substituted in.
- Page 13, Exercise 8

**Elementary Operations**

- Two systems of linear equations that have the same solution set are **equivalent**
- Note that the system given by

\[
\begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 \\
0 & 0 & 1 & 1
\end{bmatrix}
\]

also has the solution $(1, 1, 1)$. The same as the “Preliminary Problem,” so these systems are equivalent.

- You can perform the following operations for a matrix and have an equivalent solution set
  - Interchange two rows
  - Multiply a row by a nonzero scalar
  - Add a constant multiple of one row to another row

We will go into details on how to do this in Section 1.2