Section 5.5 – Substitution

A Look Back:

- Recall the chain rule, where for example \( \frac{d}{dx} e^{x^2} = e^{x^2} (2x) \).

- Clearly, since we know integrals undo derivatives, we would have \( \int e^{x^2} (2x) \, dx = e^{x^2} + c \).
  
  If we let \( u = x^2 \), we would get \( \frac{du}{dx} = 2x \), or \( du = 2x \, dx \).

  Substituting \( u \) and \( du \) into the integral, we would have \( \int e^u \, du = e^u + c = e^{x^2} + c \).

- This ‘method’ is the chain rule used backwards.

- You CANNOT evaluate an integral of a product (like above) without using substitution.

- Formally, the definition of the substitution rule is \( \int f(g(x)) \, g'(x) \, dx = \int f(u) \, du \) where \( u = g(x) \).

Substitutions with Constants:

- If \( g(x) = cx \), then setting \( u = cx \) yields \( \frac{du}{dx} = c \), or \( dx = \frac{du}{c} \).

  **Example.** Evaluate \( \int e^{2x} \, dx \)

Other Substitutions Require More Pieces:

- **Example.** Evaluate \( \int \frac{x}{(x^2 + 1)^2} \, dx \)
• Example. Evaluate $\int y^3 \sqrt{2y^4 - 1} \, dy$


• Example. Evaluate $\int \frac{x^2}{\sqrt{1-x}} \, dx$

Definite Integrals:
• So long as you change back to $x$, these are no different than the above problems.

• Example. Evaluate $\int_{\sqrt{\pi}}^{\pi} x \cos(x^2) \, dx$
Average Value:

- We define the average value of $f$ on the interval $[a,b]$ as $\frac{1}{b-a} \int_a^b f(x) \, dx$.

- Example. Find the average value of $f(x) = e^{2x}$ on $[1,2]$. 