Section 4.7 – Antiderivatives

A Recap of Some Derivatives:

- Let’s recall some derivatives we have seen so far…
  \[ \frac{d}{dx} \sin x = \cos x \]
  \[ \frac{d}{dx} \ln x = \frac{1}{x} \]
  \[ \frac{d}{dx} e^{4x} = 4e^{4x} \]
  \[ \frac{d}{dx} \cos(2x^2 + x) = -(4x + 1) \sin(2x^2 + x) \]

- If we wanted to ‘undo’ the derivative, we could say
  Antiderivative[ \cos x ] = \sin x
  Antiderivative[ \frac{1}{x} ] = \ln x
  Antiderivative[ 4e^{4x} ] = e^{4x}
  Antiderivative[ -(4x + 1) \sin(2x^2 + x) ] = \cos(2x^2 + x)

- In this section we will not focus much on finding this antiderivative, we will just be identifying the relationship between them.

The Arbitrary Constant:

- Look at the functions below with their derivatives, what do you notice?
  \[ \frac{d}{dx} (\sin x + 17) = \cos x \]
  \[ \frac{d}{dx} (\sin x - 15) = \cos x \]
  \[ \frac{d}{dx} (12 + \ln x) = \frac{1}{x} \]
  \[ \frac{d}{dx} (e + \ln x) = \frac{1}{x} \]
  \[ \frac{d}{dx} (e^{4x} - 7) = 4e^{4x} \]
  \[ \frac{d}{dx} (e^{4x} + 8) = 4e^{4x} \]

- Adding (or subtracting) a constant to a function only shifts it up or down. It does not affect the derivative (or shape) of that function.
• In that regard, any time you take the antiderivative of a function, you end up with what is called an arbitrary constant:

Antiderivative[\cos x] = \sin x + c

Antiderivative\left[\frac{1}{x}\right] = \ln x + c

Antiderivative[4e^{4x}] = e^{4x} + c

Antiderivative[-(4x + 1)\sin(2x^2 + x)] = \cos(2x^2 + x) + c

The “Easy” Antiderivative:

• Recall \frac{d}{dx} x^n = n \cdot x^{n-1}

• Shifting the value of \(n\) by 1 we see \frac{d}{dx} x^{n+1} = (n + 1) \cdot x^n

• Dividing by \(n + 1\) we find \frac{d}{dx} \left(\frac{x^{n+1}}{n + 1}\right) = x^n

• So the antiderivative of a function to a power is given by

\text{Antiderivative}[x^n] = \frac{x^{n+1}}{n + 1} + c

• Example. Find the antiderivative of \(f'(x) = 2x^2 - 3x + 7\) (find \(f\))

• Example. Find the antiderivative of \(f'(x) = 5x^3 + 8x^2 + 7x\) (find \(f\))

• Don’t forget the arbitrary constant!

• Q: What value of \(n\) will NOT work in the formula above? Have we seen it already?

A: ____________________________

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