Section 2.6 – Implicit Differentiation

More Limitations:

• So far we have seen only functions defined explicitly in terms of one variable. For example, 
  \[ f(x) = x \cdot \cos(x^2) \]

• Some functions are not defined explicitly, and the second variable (the \( f \) in the above case) is hidden inside the relation itself. That is \( f^2 + 2x = \cos x \). These are called ________________.

• It is still advantageous to find the derivative of such relations, without solving for the variable explicitly.

The Chain Rule in Disguise:

• What you need to remember is that in the example above, \( f \) is actually a function, and should be treated as such. Which means you need to use the chain rule when differentiating it.

• For example, let’s look at the problem below with several different \( y(x) \) functions defined in the [ ] and see if we can see the pattern:
  \[
  \begin{align*}
  \left( [x^2 + 3x]^2 + 5x = 2[x^2 + 3x] \right)' & \Rightarrow 2[x^2 + 3x]'(x^2 + 3x)' + 5 = 2\left( x^2 + 3x \right)' \\
  \left( [e^{2x}]^2 + 5x = 2[e^{2x}] \right)' & \Rightarrow 2[e^{2x}]'(e^{2x})' + 5 = 2\left( e^{2x} \right)'
  \end{align*}
  \]

• To find the derivative of an implicit relation:
  1. Differentiate the entire line as you would any other function, keeping the = sign in place
  2. Be sure to use the chain rule!
  3. Solve for the derivative (\( f', y' \), etc.)

• Example. Find \( y' \) if \( x + y = 3 \)
Example. Find $y'$ if $x^2y + \cos(x) = 3$

Example. Find $y'$ if $x^2y^3 + \cos(y) = 3$

Example. Find the derivative of the function $x^2 + 2xy - y^2 + x = 2$ at $(1,2)$