Section 2.2 – The Derivative as a Function

The Derivative as a Function:

- Recall that we defined \( f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \).
- Of course, this limit must exist to work properly.
- If the limit exists at \( x = a \), then the function is said to be \( \underline{\text{continuous}} \) at \( x = a \).
- If the limit exists on some interval (specifically, for all values within that interval) then the function is said to be \( \underline{\text{differentiable}} \).
- If the limit exists for all values in its domain, then the function is said to be \( \underline{\text{differentiable}} \).
  This is the same concept as continuous, where if no point or interval is given, you assume it means everywhere.
- So long as the limit definition above exists, it can be interpreted as a function itself, and it is useful to help us understand the behavior of \( f(x) \).
- There are several other symbols used for the derivative of a function, including \( f'(x) = \frac{df}{dx} = \frac{d}{dx} f(x) = Df(x) = D_x f(x) \).
- Of course, if the function is not \( f \), say it is \( y(x) \), then the above definitions would change accordingly.

Critical Points:

- Q: What do you notice about the tangent lines given in the above picture?
  A: \( \underline{\text{The tangent lines are horizontal at the critical points.}} \).
- Q: What is so special about these points?
  A: \( \underline{\text{The critical points are where the derivative is zero or undefined.}} \).

- We will cover this in more detail later, but the above points are examples of \( \underline{\text{critical points}} \), or points where the derivative is zero or undefined. Above, these critical points have a first derivative of zero.
- You can ‘match’ a graph of a function with its derivative by looking at the critical points, whenever you have a critical point on your original function, the derivative function will be zero (or undefined) there.
• Q: Can you identify which is the original function and which is the derivative?
  A: ________________________________

  ![Graph of a function and its derivative]

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  A: ________________________________

  ![Graph of a function and its derivative]

**Sign of the derivative:**

- When a function is increasing, the slope of the tangent lines will be positive, hence the derivative is ____________ when a function is ____________.
- In a similar manner, the derivative is ____________ when a function is ____________.
- When a function changes from increasing to decreasing, you have a ____________, where the derivative ________________.
What Can Go Wrong:

- Recall the requirements for continuity.
- If a function is not continuous, then it cannot be differentiable. So the following is not differentiable at the values $a$, $b$ and $c$.
- Differentiability is actually more strict than continuity.
- Any sharp corners will not be differentiable.
- Also, there may be places where the function has a vertical tangent.