Section 2.1 – Derivatives and Rates of Change

Recap:

- Let's look at the function \( f(x) = -x^2 + 2x + 3 \).

Q: What is the equation of the line through the points on the graph at \( x = 2 \) and \( x = 4 \)?
A: 

Q: What about the equation between 2 and 3?
A: 

Q: Can you predict the tangent line at \( x = 2 \)?
A: 

The General Process:

- From an original starting point \( (a, f(a)) \) we are looking at the slope of the line connecting it to another point a distance of \( h \) away from \( a \).

- What is the slope of this dashed line connecting \( (a, f(a)) \) and \( (a+h, f(a+h)) \)?

\[
\begin{align*}
\Delta y &= f(a+h) - f(a) \\
\Delta x &= (a+h) - a = h \\
slope &= \frac{\Delta y}{\Delta x} = \frac{f(a+h) - f(a)}{h}
\end{align*}
\]
• We want to let \( h \) tend to zero, so that the point \((a+h, f(a+h))\) collapses into \((a, f(a))\). So we take 
\[
\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.
\]
• This is slope of the tangent line, also called the instantaneous rate of change of \( f(x) \) at \( x = a \).

**Same Definition, Different Names:**

• The formula given by \( \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \) has a few different names.

• The ____________ to the curve \( f(x) \) at the point \( a \).

• The ____________ of the function \( f(x) \) at the point \( a \).

• The ____________ of \( f(x) \) at the point \( a \).

• The ____________ of \( f(x) \) at the point \( a \).

• If \( f(x) \) represents distance at any time, then the above definition is the ____________ at the point \( a \). If we take the absolute value of this, it represents ____________.

**Some Examples:**

• *Find the slope of the tangent line to the parabola \( y = x^2 - 3x \) at the point \( x = -1 \).*
• Find an equation of the tangent line to the curve \( y = \sqrt{5x-4} \) at the point \( x = 4 \).

• Find the slope of the tangent to the curve \( y = \frac{1}{x-3} \) at any point \( a \).
Find the slope of the tangent to the curve \( y = 2\sqrt{x} \). This time we will leave it as \( x \) instead of \( a \).