Section 5.1 – Systems of Equations in Two Variables

- When you have two or more equations that need to be solved at the same time, we refer to them as a **system of equations**
- Count the number of equations and count the number of variables. In general, the system can be solved uniquely if there are the same number of each
- This first section focuses on finding the solution to a system of two linear equations
- Is it possible there is no solution (inconsistent system), or there are many solutions (non-unique solutions)?
  - Yes. When finding the intersection point of two lines, it is possible the lines will not intersect. **How can this happen?**
  - There can also be many solutions if the lines are the same (one is on top of the other)

**Solving A System of Linear Equations Graphically:**

- This is not the best way, because you have to rely on your scale being exact, and your graphing abilities being exact
- This does help you find an approximate solution, and help you determine if there is a solution at all

**The Substitution Method:**

- If you have one equation that is already solved for $x$ or $y$, the substitution method is appropriate
- In general, for the system
  \[
  y = m_1x + b_1 \\
  y = m_2x + b
  \]
  1. Plug the first equation into the second, $m_1x + b_1 = m_2x + b$
  2. You should have an equation with only one variable ($x$ or $y$). Solve for that variable
  3. Plug that value into either one of the original equations and solve for the other variable

**Example.** Solve $x - y = -5, \ x = -4y$
  Notice the second equation is solved for $x$
  \[
  (-4y) - y = -5 \quad \text{(step 1, plug this into the first equation)}
  \]
  \[
  -5y = -5, \ y = 1 \quad \text{(step 2, solve for $y$)}
  \]
  \[
  x = -4(1) = -4 \quad \text{(step 3, solve for $x$)}
  \]
  The solution is ($-4, 1$)

- Note that this method also works well when you have one equation that is 'almost' solved

**Example.** Solve $2x - y = 1, \ x + 2y = -7$
  Notice the first equation is 'almost' solved for $y$, that is, $y = 2x - 1$ (so is the other, really)
  Let's finish the steps…
  \[
  x + 2(2x - 1) = -7
  \]
  \[
  x + 4x - 2 = -7
  \]
  \[
  5x = -5, \ x = -1
  \]
  And $y = 2(-1) - 1 = -3$
  Solution: ($-1, -3$)
  Check: $2(-1) - (-3) = 1, \ (-1) + 2(-3) = -7$. Yes
The Elimination Method:

- The elimination method works well when both the equations have coefficients in front of the x and y terms
- In general, for the system
  \[ a_1y + b_1x = c_1 \]
  \[ a_2y + b_2x = c_2 \]
  1. You want to eliminate either the x or y term
  2. First, get the coefficients (a's or b's) to have the same number and opposite signs
  3. Add the two equations, combine like terms
  4. You should have an equation with only one variable (x or y). Solve for that variable
  3. Plug that value into either one of the original equations and solve for the other variable

Example. Solve \( 3x + 4y = -2, \ -3x - 5y = 1 \)

Notice it would not be 'easy' to solve for either variable in either equation
It is already set up that the x's coefficients have the same number and opposite signs
Add these equations together
\[
3x + 4y = -2 \\
-3x - 5y = 1
\]
\[
- y = -1, \quad y = 1
\]
\[
3x + 4(1) = -2, \quad -3x - 5(1) = 1 \quad \Rightarrow x = -2 \quad \text{(Note that using both is a way to check)}
\]
The solution is \((-2, 1)\)

Example. Solve \( 3x + 4y = 6, \ 2x + 3y = 5 \)

Notice it would not be 'easy' to solve for either variable in either equation
Let's eliminate the x. The least common multiple of 3 and 2 is 6
Multiply the first equation by –2 to get \(-6x - 8y = -12\)
And multiply the second equation by 3 to get \(6x + 9y = 15\)
Add these equations together
\[
-6x - 8y = -12 \\
6x + 9y = 15
\]
\[
1y = 3 \\
3x + 4(3) = 6 \quad \text{or} \quad 2x + 3(3) = 5 \quad \Rightarrow x = -2 \quad \text{(Note that using both is a way to check)}
\]
The solution is \((-2, 3)\)

What Can Go Wrong?

Example. Solve \( 4x - 2y = 5, \ 6x - 3y = -10 \)

Notice it would not be 'easy' to solve for either variable in either equation. Use elimination
The least common multiple of 4 and 6 is 12
Multiply the first equation by –3 to get \(-12x + 6y = -15\)
And multiply the second equation by 2 to get \(12x - 6y = -20\)
Add these equations together
\[
-12x + 6y = -15 \\
12x - 6y = -20 \\
0 = -35
\]
No \((x, y)\) satisfy the system (no solution)
Example. Solve \(12x - 3y = 6, \ 8x - 2y = 4\)

Use elimination or substitution by solving one equation

Let's solve the first equation by dividing everything by 3 and solving for \(y\):
\[y = 4x - 2\]

Sub this into the second equation:
\[8x - 2(4x - 2) = 4\]

Solve this equation for \(x\):
\[8x - 8x + 4 = 4\]
\[4 = 4\]

This will always be true. All \((x, y)\) satisfy the system