Section 4.5 – Solving Exponential and Log Equations

Solving Exponential Equations:

- Equations with variables in the exponent are called **exponential equations**
- If the base is the same on both sides of the equation, you can just equate the exponents
- **Example.** Solve \( 2^x = 32 \)
  \[ 2^x = 2^5 \quad \Rightarrow \quad x = 5 \]
- **Example.** Solve \( 3^{x+4x} = \frac{1}{27} \)
  \[ 3^{2x} = 3^{-3} \]
  \[ x^2 + 4x + 3 = 0, \quad x = -3, -1 \]
- Trouble comes when the base is not the same. You will then have to solve the equation using logs
  - **Example.** Solve \( 2^x = 40 \)
    \[ \ln(2^x) = \ln(40) \]
    \[ x = \frac{\ln 40}{\ln 2} \]
  - **Example.** Solve \( 250 - (1.87)^x = 0 \)
    \[ 250 = (1.87)^x \]
    \[ x = \frac{\ln 250}{\ln 1.87} \]
  - **Example.** Solve \( e^x - 6e^{-x} = 1 \)
    \[ e^{2x} - e^x - 6 = 0 \]
    \[ (e^x - 3)(e^x + 2) = 0 \]
    \[ e^x = 3, \quad x = \ln 3 \]
    \[ e^x = -2 \quad \text{(never)} \]

Solving Log Equations:

- Often it is useful to change to exponential in form
- **Example.** Solve \( \log_2 x = -3 \)
  \[ 2^{-3} = x \quad \Rightarrow \quad x = 1/8 \]
- **Example.** Solve \( \log_5(8 - 7x) = 3 \)
  \[ 5^3 = 8 - 7x \]
  \[ 7x = -117 \quad \Rightarrow \quad x = -117/7 \]
- **Example.** Solve \( \log x - \log(x + 3) = -1 \)
  \[ \log \frac{x}{x+3} = -1 \]
  \[ 10^{-1} = \frac{x}{x+3} \]
  \[ 0.1(x+3) = x \quad \Rightarrow \quad 0.9x = 0.3 \quad \Rightarrow \quad x = 1/3 \]