Section 1.7 – Symmetry

• Being able to see the symmetry in functions helps to analyze their behavior
• We can move a point \((a,b)\) located in Quadrant I to any other quadrant by reflecting it…

\((a,b)\) reflected in the \(x\) axis will result in the point \((a,-b)\)
\((a,b)\) reflected in the \(y\) axis will result in the point \((-a,b)\)
\((a,b)\) reflected through the \textit{origin} will result in the point \((-a,-b)\)

• We define symmetry based on these reflections. If any point \((x,y)\) on a graph has a corresponding point \((x,-y)\) then the graph is said to be \textit{symmetric with respect to the \(x\) axis}. In other words, above and below the \(x\) axis are mirror images of each other
• If any point \((x,y)\) on a graph has a corresponding point \((-x,y)\) then the graph is said to be \textit{symmetric with respect to the \(y\) axis}. In other words, the left and right of the \(y\) axis are mirror images of each other
• If any point \((x,y)\) on a graph has a corresponding point \((-x,-y)\) then the graph is said to be \textit{symmetric with respect to the origin}. In other words, the graph is the same if we rotate it 180 degrees (turn it upside down)

\textit{Example, page 163 (page 153 version 2) numbers 2, 4 and 6 (see photos)}
Symmetric with respect to \(x\) axis: 2 no, 4 no, 6 yes
Symmetric with respect to \(y\) axis: 2 yes, 4 no, 6 yes
Symmetric with respect to the origin: 2 no, 4 no, 6 yes

\textit{Even and Odd Functions:}
• If a function, \(f\), is symmetric with respect to the \(y\) axis, it is considered to be an \textbf{even function}. It can be tested by the fact that for every \(x\) in the domain of \(f\), \(f(x) = f(-x)\). \textit{How does this relate to the definition above?}
• If a function, \(f\), is symmetric with respect to the \textit{origin}, it is considered to be an \textbf{odd function}. It can be tested by the fact that for every \(x\) in the domain of \(f\), \(f(-x) = -f(x)\). \textit{How does this relate to the definition above?}
• To determine if a function is even or odd (without graphing), analyze \(f(-x)\)

\textit{Example. Determine even/odd for } \(f(x) = x + \frac{1}{x}\)

\[f(-x) = -x - \frac{1}{x} = -\left(x + \frac{1}{x}\right) = -f(x).\] \(\text{So } f \text{ is odd}\)