Section 1.2 – Functions and Graphs

- A relation is a correspondence (or relationship) between one set of values and another.
- Last section (1.1) we discussed the relation between Celsius and Fahrenheit. We described it as a linear relationship, we looked at the graph and some ordered pairs.
- It was mentioned that putting Celsius on the horizontal axis was arbitrary. Why? Why did we not distinguish between the dependent and independent variables?
- It is standard to put the independent variable along the horizontal. If your grades depend on the number of hours you study, which is the dependent variable and which is the independent variable?
- A function is a relationship between two variables, but the dependent variable is unique. In other words, for every \( x \) (independent value), there is one and only one \( y \) (dependent value).
- So a function is a relation. Is a relation a function?
- If you have a picture of the relation, you can use the vertical line test. Since you can only have one \( y \) for every one \( x \), a vertical line can pass through a function only once. If it passes more than once it is not a function. Which of the below relations below are functions and which are not?

![Relations](image)

- Independent variables are collectively called the domain and the dependent variables are collectively called the range.
- Domain is where you start from. Put those values into the function (or relation), and what you get out makes up your range.

![Diagram](image)

- Say \( f \) is a function of \( x \). Which is the dependent variable and which is the independent variable? What values make up the range?
- We use a particular kind of notation for functions. In the above case, for \( f \) a function of \( x \), we say \( f(x) \). This does not mean \( f \) times \( x \)! It is a quick way of saying \( f \) is a function of \( x \).
- Example. A set of members of a rock band \( \rightarrow \) an instrument each person plays \( \rightarrow \) a set of instruments
  - The domain is the band members, the range is the instruments. Can any member of the band play more than one instrument? Yes, so this is not a function.
- Remember, the easy test for whether a relation is a function is that for every element in the domain, there can be only one element in the range. Every \( x \) maps to only one \( y \). But, can one \( y \) map to two different \( x \)'s?
Finding Values in the Range:

- Given a function \( f(x) \) to determine values in the range, you need only input what is in parenthesis in for \( x \)
- **Example.** For \( f(x) = 5x^2 + 4x \), find
  
  \[
  
  f(0) = 5(0)^2 + 4(0) = 0 \\
  f(-1) = 5(-1)^2 + 4(-1) = 5 - 4 = 1 \\
  f(3) = 5(3)^2 + 4(3) = 45 + 12 = 57 \\
  f(t) = 5t^2 + 4t \\
  f(t-1) = 5(t-1)^2 + 4(t-1) = 5(t^2 - 2t + 1) + 4t - 4 = 5t^2 - 6t + 1
  
  

Domain Restrictions:

- The domain of a function can be restricted, which could be given directly or implied
- When a domain is given directly it would be of a form such as \( f(x) = 2x + 1 \ {\{x \geq 0}\} \). The style may vary, but the point is somehow made that \( x \) can only take certain values
- The domain of a function can be implied by the fact that some \( x \) values are not allowed. Because the Cartesian plane takes only real values, the most obvious domain restriction is the potential division by zero or square root of a negative
- There could be domain restrictions if the function is or has:
  - a rational expression (a polynomial in the denominator)
  - a square root expression (or any other even root expression)
- **Examples of this would be**
  
  \[
  
  f(x) = \frac{2x-1}{x} \\
  g(x) = \sqrt{x+1} - 12x \\
  h(x) = \frac{1}{\sqrt{2-x}} + x^2
  
  
  
  We must restrict the domain so these values are not included. In other words, the domain restrictions are as follows:
  
  for \( f(x) \), \( x \neq 0 \)
  
  for \( g(x) \), \( x+1 \geq 0 \), or \( x \geq -1 \)
  
  for \( h(x) \), \( 2 - x > 0 \), or \( x < 2 \)

- **Example.** Find the domain for \( f(x) = \frac{1}{x^4} \)
  
  \( x \neq 0 \)

- **Example.** Find the domain for \( f(x) = \frac{1}{3x^2 - 10x - 8} \)
  
  \[
  
  3x^2 - 10x - 8 = (3x + 2)(x - 4) \\
  x \neq 4, -\frac{2}{3}
  
  