A rational expression is the quotient of two polynomials.

Behaviors of Rational Expressions:
- The domain of an expression is the set of all real numbers for which the expression is defined. It is all the values 'allowed' to be substituted in for the variable.
- The domain of a rational expression may be restricted, because you can never divide by zero. Any place that makes the denominator of the rational expression go to zero is restricted.
- Example. Find the domain of \( \frac{15x - 10}{2x(3x - 2)} \)
  \[ 2x(3x - 2) \neq 0 \Rightarrow x \neq 0, \frac{2}{3} \]
- A rational expression \( \frac{p(x)}{q(x)} \) will be equal to zero when \( p(x) = 0 \), provided that \( q(x) \neq 0 \) at the same time.
- Example. Find when \( \frac{15x - 10}{2x(3x - 2)} = 0 \)
  \[ 15x - 10 = 0 \Rightarrow x = \frac{2}{3} \]
  but \( 2(2/3)\left(3 \cdot \frac{2}{3} - 2\right) = 0 \)
  So there is no solution.
- Example. Find when \( \frac{x}{x^2 - 4} = 0 \)
  \[ x = 0 \]
  and \( 0^2 - 4 \neq 0 \)
  So the solution is \( x = 0 \).

Working with Rational Expressions:
- Example. Operate and simplify \( \frac{r - s}{r + s} \cdot \frac{r^2 - s^2}{(r - s)^2} \)
  \[ \frac{1}{r + s} \cdot \frac{(r - s)(r + s)}{r - s} = 1 \]
- Example. Operate and simplify \( \frac{a^2 - a - 2}{a^2 - a + 6} \div \frac{a^2 - 2a}{2a + a^2} \)
  \[ \frac{a^2 - a - 2, 2a + a^2}{a^2 - a - 6, a^2 - 2a} = \frac{(a - 2)(a + 1), a(2 + a)}{a - 3(a + 2), a(a - 2)} = a + 1 \]
• When adding and subtracting rational expressions, you must find the least common denominator, just like you do when you add or subtract fractions

- Example. Operate and simplify \( \frac{a}{a-b} + \frac{b}{b-a} \)

\[
\frac{a}{a-b} - \frac{b}{a-b} = \frac{a-b}{a-b} = 1
\]

- Example. Operate and simplify \( \frac{x-1}{x-2} - \frac{x+1}{x+2} - \frac{x-6}{4-x^2} \)

\[
\frac{(x-1)(x+2)}{x^2-4} - \frac{(x+1)(x-2)}{x^2-4} - \frac{x-6}{x^2-4} = \frac{(x^2+x-2)-(x^2-x-2)+(x-6)}{x^2-4}
\]

\[
= \frac{3x-6}{x^2-4} = \frac{3(x-2)}{(x-2)(x+2)} = \frac{3}{x+2}
\]

• A complex rational expression is when you have a rational expression inside another rational expression. It is similar in form to a compound fraction (fraction inside a fraction)

- Example. Simplify \( \frac{a^2+b^2}{a^2-ab+b^2} \)

\[
\frac{ab}{a^2-ab+b^2} + \frac{b^2}{a^2-ab+b^2} = \frac{a^3+b^3}{ab(a^2-ab+b^2)} = \frac{(a+b)(a^2-ab+b^2)}{ab(a^2-ab+b^2)} = \frac{a+b}{ab}
\]

- Example. Simplify \( \frac{x}{1-x} + \frac{x}{1+x} \)

\[
\frac{x}{x} + \frac{1+x}{1-x} \cdot \frac{x}{x} = \frac{(1-x)(1+x)(1+x)+x^2}{x(1+x)(1-x)} \cdot \frac{(1-x)(1+x)(1-x)+x^2}{x(1+x)(1-x)} = \frac{1-x}{1+x}
\]