Section R.3 – Polynomials

• A polynomial in one variable is an expression of the form \( a_nx^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0 \), where the \( a_i \)'s are real number coefficients. For nonzero \( a_n \), the expression is said to be of \( n \)th degree (the highest power is \( n \)) and the leading coefficient is \( a_n \).

• Example. Determine the terms and degree of \( 2m^3 - m^2 - 4m + 11 \)
  
  It has terms: \( 2m^3, -m^2, -4m, 11 \)
  
  The leading coefficient is 2
  
  The degree is 3

• You can have a polynomial in several variables, and the degree would be the highest sum of exponents.

• Example. Determine the terms and degree of \( 6p^3q^2 - p^2q^4 - 3pp^2 + 5 \)
  
  The terms are: \( 6p^3q^2, -p^2q^4, -3pp^2, 5 \)
  
  The degree is 6

• When working with polynomials, you can combine like terms. Be sure if multiplying polynomials you account for all terms in the product

  i.e. \((A + B)(A + B) = A^2 + 2AB + B^2\)

• Example. Operate on \((5x^2 + 4xy - 3y^2 + 2) - (9x^2 - 4xy + 2y^2 - 1)\)

  \[ 5x^2 + 4xy - 3y^2 + 2 - 9x^2 + 4xy - 2y^2 + 1 = -4x^2 - 5y^2 + 8xy + 3 \]

• Example. Compute \((5x + 2y + 3)(5x + 2y - 3)\)

  \[ 5x(5x + 2y - 3) + 2y(5x + 2y - 3) + 3(5x + 2y - 3) \]

  \[ = 25x^2 + 10xy - 15x + 10xy + 4y^2 - 6y + 15x + 6y - 9 \]

  \[ = 25x^2 + 20xy + 4y^2 - 9 \]