Section 2 – Interest

• **Interest** is the dollar amount paid to you for lending someone money (or the amount you pay to someone for borrowing money)

• **Principal** is the amount of money borrowed or deposited.

• **Interest rate** is the percentage charged for borrowing a principal amount (or the percentage deposited for depositing a principal amount). In the calculations you will always convert this to a decimal.

**Simple Interest:**

• **Simple interest** is earned only on the principal (you do not earn anything on the interest earned over time).

  *Formula*: \( \text{Interest} = \text{Principal} \times \text{Rate} \times \text{Time} \)

  • **Example**: You deposit $3000 in a savings account which earns 5%. Find the simple interest at the end of the first year

    \[ \text{Solution: Interest} = P \cdot R \cdot T = 3000 \times 0.05 \times 1 = 150 \]

  • **Example**: A loan of $2040 has been made at 7.5% for four months. Find the interest and maturity value.

    \[ \text{Solution: Interest} = P \cdot R \cdot T = 2040 \times 0.075 \times (4/12) = 51 \]

    Maturity Value = original + interest = 2040 + 51 = 2091

**Compound Interest:**

• **Compound interest** is computed on the original principal as well as the interest earned (in other words, when you compound, you earn interest on your interest).

• How often it is compounded depends… it can be yearly (1 time per year), monthly (12 times per year), semiannually (2 times per year), etc.

• **NOTE**: you want to keep as many decimal places in the computation as possible.

• Almost all lenders are required to provide you with the **Annual Percentage Rate (APR)** because it can be confusing when you start to compound different rates at different times.

• The **APR** is found with the number of times compounded \( n \) and the interest rate \( r \) as below:

  \[ \text{APR} = \left(1 + \frac{r}{n}\right)^n - 1 \]

• For example, if you compound semiannually at 5% \( r \), this means you compound twice in one year \( n = 2 \), which means your **APR** is

  \[ \text{APR} = \left(1 + \frac{0.05}{2}\right)^2 - 1 = 1.050625 - 1 = 0.050625 \text{ or } 5.0625\% \]

• If you compound monthly at 5%, you have an **APR** of

  \[ \text{APR} = \left(1 + \frac{0.05}{12}\right)^{12} - 1 \approx 0.05116 \]

• The formula for total amount of money \( A \) given principal \( P \) invested for \( t \) years at a fixed **APR** is:

  \[ A = P \cdot (1 + \text{APR})^t \]

• **Does this formula look familiar?**
• First let’s go to the practice problems to make sure we can use our calculator to compute this value.

• **Example:** You deposit $4200 into an account earning 4% compounded quarterly. What is the APR? How much interest will you earn after 10 years? How much will you have?

  **Solution:**
  
  $P = 4200$, $r = 0.04$, $t = 10$, $n = 4$
  
  $APR = \left( 1 + \frac{0.04}{4} \right)^4 - 1 \approx 0.04060401$
  
  $A = 4200 \times (1.04060401)^{10} = 6253.23$ (this is how much you will have after 10 years)
  
  You have earned $6253.23 - 4200 = 2053.23$

**Comparing Account Types:**

• It is not always the higher rate that determines which is investment is better, the number of times you compound also affects the amount you earn.

• When investing, whichever has the highest APR is best! When borrowing, the lower APR is best!

**How Much to Invest:**

• You can use the formula for compound interest to determine how much should be invested to earn a certain amount in a certain time period.

• **Example:** What is the APR if you earn 6% compounded yearly? How much money should be deposited in that account today so it will be $1,000,000 in 20 years?

  **Solution:**
  
  $APR = \left( 1 + \frac{r}{n} \right)^n - 1 = \left( 1 + \frac{0.06}{1} \right)^{10} - 1 = 0.06$
  
  $P$ is unknown, $A = 1,000,000$, and $r = 0.06$
  
  $P = A \times (1 + APR)^t = 1,000,000 \times (1.06)^{20} = 311,804.72$
  
  You should invest at least $311,804.72$

• **Example:** For the same problem above, what if you have 30 years?

  **Solution:** $P$ is unknown, $A = 1,000,000$, $r = 0.06$ and $n = 1$

  $P = A \times (1 + APR)^t = 1,000,000 \times (1.06)^{30} = 174,110.13$