Section 5 –Integers with Exponents

Positive Exponents:
• Let $a$ be any integer and $m$ be any nonzero whole number. Then $a^m = a \cdot a \cdot a \cdot \ldots \cdot a$ ($m$ times)
• Example: $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$
• Example: $(-1)^3 = (-1) \cdot (-1) \cdot (-1) = -1$
• Property 1: $a^m a^n = a^{m+n}$
  Example: $3^2 \cdot 3^4 = (3 \cdot 3)(3 \cdot 3 \cdot 3 \cdot 3) = 3^6$
• Property 2: $(ab)^m = a^m b^m$
  Example: $(3 \cdot 4)^2 = (3 \cdot 4)(3 \cdot 4) = (3 \cdot 3)(4 \cdot 4) = 3^2 \cdot 4^2$
• Property 3: $(a^m)^n = a^{mn}$
  Example: $(3^2)^4 = (3^2)(3^2)(3^2) = a^{24}$
• Property 4: $a^0 = 1$ ($a \neq 0$)

Examples:
• Simplify $5 \cdot 6 \cdot 5 \cdot 5 \cdot 6 \cdot 6$
  $5 \cdot 6 \cdot 5 \cdot 5 \cdot 6 \cdot 6 = 5^3 \cdot 6^3 = (5 \cdot 6)^3$
• Expand $(7 \cdot 5)^3$
  $7 \cdot 7 \cdot 7 \cdot 5 \cdot 5 \cdot 5$
• Rewrite $9^2 \cdot 12^3 \cdot 2$ with a single exponent
  $(3^2)^2 \cdot (3^2)^3 \cdot 2 = 3^4 \cdot 3^3 \cdot 2^4 \cdot 2 = 3^7 \cdot 2^7 = (3 \cdot 2)^7 = 6^7$
• Use your calculator to evaluate $3 \cdot 5^4$
  $1875$

Negative Exponents:
• Negative exponents are equivalent to fractions. When an exponent is negative, it means the value shifts from the numerator to the denominator (or vice versa).
• Examples:
  $3^{-1} = \frac{3^{-1}}{1} = \frac{1}{3^1}$
  $\frac{2^{-4}}{3} = \frac{1}{3 \cdot 2^4} = \frac{1}{48}$
  $\frac{3}{5^{-2}} = \frac{3 \cdot 5^2}{1} = 3(25) = 75$
• We can now add another Property, which is $\frac{a^m}{a^n} = a^{m-n} = a^{m+n}$
• For example, $\frac{5^7}{5^6} = 5^{7-6} = 5^1 = 5$