Section 3 – Multiplication and Division of Integers

Different Approaches to Multiplication:

- The notation for multiplication \((a \times b)\) has several symbol types:
  \(a \cdot b\), \(a \times b\), \(a(b)\), \((a)b\), \((a)(b)\)

- If we are expressing multiplication of non-numeric values (i.e. \(a \cdot b\)) we often shorthand and write just \(ab\). Obviously we can do this with numbers.

- **Repeated addition**: For any whole numbers \(a\) and \(b\) (\(a \neq 0\)), then \(a \cdot b = b + b + b + \ldots + b\) (\(a\) times)

- For example, \(3 \cdot 5 = 3 + 3 + 3 + 3 + 3 = 15\)

- **Rectangular array approach**: \(a \cdot b\) is the number of elements in a rectangular array with \(a\) rows and \(b\) columns

- For example, \(3 \cdot 5 = 15\) because it is equivalent to the number of elements in the array with 3 rows and 5 columns

- **Number line approach**: \(a \cdot b\) is the same as starting from zero (facing the positive numbers), and taking \(a\) steps of length \(b\).

- For example \(2 \cdot 5\) is shown below with 2 steps of length 5

- Probably the most important thing to remember is the sign of the result. If we take the repeated addition approach, \((-3) \cdot 5 = (-3) + (-3) + (-3) + (-3) + (-3) = -15\).

- Note that \(5 \cdot (-3)\) thought of with number line approach indicates 5 steps of length –3. To measure out a negative length we must turn around. So we are now at zero, facing left and take 5 steps of length 3 which brings us to \(-15\).

- In a similar fashion, \((-3) \cdot 5\) can be thought of as 3 BACKWARD steps of length 5.

- So clearly if only one of the values is negative the result will be negative. And we saw earlier that if both of the values were positive, the result would be positive.
• But what about a negative times a negative? The only way to visualize this, really, is to use the number line approach. So if we want to find \((-2) \cdot (-5)\), the distance \((-5)\) means we need to turn around so we are facing the negative direction. Then we take 2 steps BACKWARD of length 5.

• Notice that the result of a **negative times a negative is positive**. You need not go through this illustration every time.

**Properties of Multiplication:**
- **Closure**: The product of any two whole numbers is a whole number
- **Commutative**: \(ab = ba\)
- **Associative**: \(a(bc) = (ab)c\)
- **Identity**: \(a \cdot 1 = 1 \cdot a = a\)
- **Distributive**: \(a(b + c) = ab + ac\), and \(a(b - c) = ab - ac\)
- **Example. Use the distributive property to mentally compute** \(5 \times 49\)
  \[5 \times 49 = 5 \cdot (50 - 1) = 5 \times 50 - 5 \times 1 = 250 - 5 = 245\]
- **Example. Use the distributive property to mentally compute** \(39 \cdot 102\)
  \[39 \cdot (100 + 2) = 39 \cdot 100 + 39 \cdot 2 = 3900 + (30 + 9) \cdot 2 = 3900 + 30 \cdot 2 + 9 \cdot 2 = 3900 + 60 + 18 = 3978\]
- **Multiplication of zero**: \(a \cdot 0 = 0 \cdot a = 0\)

**Division:**
- We can describe two types of division. In each case you have a number of items and a number of groups. In **partitive division** the number of groups is fixed and you want to find the number of items. In **measurement division** the number of items is fixed and you want to find the number of groups. These words (partitive and measurement) are seldom used, and we think of “to divide” as breaking into parts.
- For example: I want to split the class of 36 into groups.
  a) I want 6 groups, how many students per group? This is **partitive division**
  b) I want 9 students per group, how many groups? This is **measurement division**
  Both are examples of division.
- If \(a\) and \(b\) are any whole numbers with \(b \neq 0\), then \(a \div b = b \left\lfloor \frac{a}{b} \right\rfloor = \frac{a}{b} \equiv c\) if and only if \(a = b \cdot c\) for some whole number \(c\).
- For example, \(20 = 4 \cdot 5\), so this implies that \(20 \div 4 = 4 \left\lfloor \frac{20}{4} \right\rfloor = \frac{20}{4} \equiv 5\)
- If \(a \neq 0\) then \(0 \div a = 0\)
- For any value of \(a\), \(a \div 0\) is undefined. **Why can we never divide by zero?**
- The sign rules for multiplication apply to division.