VOLUME FORMULAS
(http://mathforum.org/dr.math/faq/formulas/)

Sphere
A three-dimensional figure with all of its points equidistant from its center.

Radius \( r \)
Diameter \( d \)
Surface area \( S \)
Volume \( V \)

\[
S = 4 \pi r^2 = \pi d^2
\]
\[
V = \frac{4}{3} \pi r^3 = \frac{1}{6} \pi d^3
\]

Ellipsoid
A three-dimensional figure all planes cross-sections of which are either ellipses or circles.

Semi-axes: \( a, b, c \) (the semi-axis is half the length of the axis, and corresponds to the radius of a sphere)

Volume \( V \)

\[
V = 4 \pi abc
\]

Sector of a Sphere
The part of a sphere between two right circular cones that have a common vertex at the center of the sphere, and a common axis. (The interior cone may have a base with zero radius.)

Radius \( r \)
Height \( h \)
Volume \( V \)

\[
S = 2 \pi rh
\]
\[
V = \frac{(\pi h^2)(2r^2 + 4rh + h^2)}{6}
\]

Prolate Spheroid
Semi-axes: \( a, b, c \) (\( a > b \))
Surface area \( S \)

\[
S = 2 \pi b(b + a \cos(e/e))
\]
where \( e = \sqrt{\left(b^2 - a^2\right)/b} \)

Oblate Spheroid
Semi-axes: \( a, b, c \) (\( a < b \))
Surface area \( S \)

\[
S = 2 \pi b(b + a \cos^{-1}(b/a)/(b/a))
\]
where \( e = \sqrt{\left(b^2 - a^2\right)/b} \)

Segment and Zone of a Sphere
Segment: the portion of a sphere cut off by two parallel planes.
Zone: the curved surface of a spherical segment

Radius of sphere \( r \)
Radius of bases \( r_1, r_2 \)
Height \( h \)
Surface area \( S \)
Volume \( V \)

\[
S = 2 \pi rh
\]
\[
V = \frac{(\pi h^2)(2r^2 + 4rh + h^2)}{6}
\]

Spherical Cap
The portion of a sphere cut off by a plane. If the height, the radius of the sphere, and the radius of the base are equal \( h = r = r_1 \), the figure is called a hemisphere.

Radius of sphere \( r \)
Radius of base \( r_1 \)
Height \( h \)
Surface area \( S \)
Volume \( V \)

\[
r = \sqrt{2} h \]
\[
S = 2 \pi rh
\]
\[
V = \frac{(\pi h^2)(2r^2 + 4rh + h^2)}{6}
\]

Frustum of a Pyramid
The portion of a pyramid that lies between the base and a plane cutting through it parallel to the base.

Height \( h \)
Area of bases \( B_1, B_2 \)
Slant height \( s \) (regular pyramid)
Perimeter of bases \( B_1, B_2 \)
Lateral surface area \( S \)
Volume \( V \)

\[
S = \frac{(B_1 + B_2)l}{2} \]
\[
V = \frac{h(B_1 + B_2 + \sqrt{B_1 B_2})}{3}
\]

Square Pyramid
The base is a square, and all triangular faces are congruent isosceles triangles.

Side of base \( a \)
Other edges \( b \)
Height \( h \)
Slant height \( s \)

Vertex angle of faces \( \alpha \)
Base angle of faces \( \theta \)
Face-to-face dihedral angle \( \beta \)
Face-to-face dihedral angle \( \phi \)

Lateral surface area \( S \)
Total surface area (including base) \( T \)
Volume \( V \)

\[
a = \sqrt{\left(b^2 - h^2\right)}
\]
\[
b = \sqrt{\left(b^2 + h^2\right)}
\]
\[
h = \sqrt{\left(b^2 + h^2\right)}
\]
\[
s = \sqrt{\left(b^2 + h^2\right)}
\]
\[
\alpha = \arccos\left(b/2a\right) = \arccos\left(b/a\right) = \arccos\left(b/\sqrt{2a}\right)
\]
\[
\beta = \arccos\left(h/2a\right) = \arccos\left(h/a\right) = \arccos\left(h/\sqrt{2a}\right)
\]
\[
\phi = \arccos\left(h^2 + 2h^2\right) = \arccos\left(h^2 / 2a^2\right) = \arccos\left(-h/\sqrt{2a}\right)
\]
\[
S = 2ah
\]
\[
T = a(2+a)
\]
\[
V = \frac{a^3h}{3}
\]
Rectangular Parallelepiped
A three-dimensional figure all of whose face angles are right angles, to all its faces are rectangles and all its dihedral angles are right angles. (A dihedral angle is an angle created by two intersecting planes.)

- Edges: a, b, c
- Diagonal d
- Total surface area (total area of all the faces of the figure): T
- Volume V

\[ d = \sqrt{a^2 + b^2 + c^2} \]
\[ T = 2(ab + ac + bc) \]
\[ V = abc \]
- Face diagonals sqrt
  \[ (a^2 + b^2) \]
  \[ (a^2 + c^2) \]

Circular Cylinder
A cylinder whose bases are circles. The line connecting the centers of the bases is called the axis.

- Height h
- Area of base: B
- Length of lateral edge: l
- Area of right section: A
- Perimeter of right section: P
- Lateral surface area: S
- Total surface area: T
- Volume: V

\[ S = 2\pi rh \]
\[ T = 2\pi rh + 2\pi B \]
\[ V = \pi r^2 h \]

Prism
A polyhedron with two congruent, parallel bases that are polygons, and all remaining faces parallelograms.

- Height: h
- Area of base: B
- Length of lateral edge: l
- Area of right section: A
- Perimeter of right section: P
- Lateral surface area: S
- Volume: V

\[ S = ph \]
\[ V = Bh = lA \]

Right Circular Cylinder
A circular cylinder in which the axis is perpendicular to the bases. (If the axis of a circular cylinder is not perpendicular to the bases, it is called an oblique circular cylinder.)

- Height: h
- Radius of base: r
- Lateral surface area: S
- Total surface area: T
- Volume: V

\[ S = 2\pi rh \]
\[ T = 2\pi rh + 2\pi r^2 \]
\[ A = B = \pi r^2 \]
\[ P = 2\pi r \]
\[ l = h \]

Tetrahedron
A three-dimensional figure with 4 equilateral triangle faces, 4 vertices, and 6 edges.

\[ v = 4, e = 6, f = 4 \]
\[ a = (\sqrt{6}/3)r \]
\[ r = (\sqrt{3}/3)r \]
\[ B = (\sqrt{6}/4)a \]
\[ S = \sqrt{3}(3)a^2 \]
\[ V = (\sqrt{2}/12)a^3 \]
\[ \text{delta} = \arccos(1/3) = 70^\circ 32' \]
\[ h = \text{height or altitude} \]
\[ h = (\sqrt{6}/3)a \]

Frustum of a Right Circular Cone
The part of a right circular cone between the base and a plane parallel to the base whose distance from the base is less than the height of the cone.

- Height: h
- Radius of bases: R, r
- Slant height: s
- Lateral surface area: S
- Total surface area: T
- Volume: V

\[ s = \sqrt{(R-r)^2 + h^2} \]
\[ S = \pi(R+r)s \]
\[ T = \pi(R+r)h + \pi(R+r)s \]
\[ V = \frac{1}{3}\pi h(R^2 + r^2 + Rr) \]

Right Circular Cone
In a right circular cone, the axis is perpendicular to the base. (If the axis of a circular cone is not perpendicular to the base, it is called an oblique circular cone.)

The height of any line segment connecting the vertex to the directrix is called the slant height of the cone.

- Height: h
- Radius of base: r
- Slant height: s
- Lateral surface area: S
- Total surface area: T
- Volume: V

\[ B = \pi r^2 \]
\[ s = \sqrt{h^2 + r^2} \]
\[ S = \pi rs \]
\[ T = \pi r(s+h) \]
\[ V = \frac{1}{3}\pi r^2 h \]

Octahedron
A three-dimensional figure with 8 equilateral triangle faces, 6 vertices, and 12 edges.

\[ v = 6, e = 12, f = 8 \]
\[ a = \sqrt{2}r \]
\[ r = (\sqrt{3}/3)r \]
\[ B = (\sqrt{2}/2)a \]
\[ r = (\sqrt{2}/3)a \]
\[ S = 2\sqrt{3}a^2 \]
\[ V = (\sqrt{2}/3)a^3 \]
\[ \text{delta} = \arccos(-1/3) = 109^\circ 28' \]