How to Sketch the Graph of a Function $f(x)$:
(types we have seen so far)

Identify the function type

1. **Algebraic**

**Root Functions** $f(x) = \sqrt[n]{g(x)}$

- Find the domain: If $n$ is even then $g(x) \geq 0$ is the domain
- Find the roots: when $g(x) = 0$
- Analyze the first and second derivatives to determine the shape
- Sketch using the critical points and intercepts

**Rational expressions** $f(x) = \frac{p(x)}{q(x)}$

- Find the domain. The point(s) $x = c$ is a domain restriction when $q(c) = 0$. If $p(c) \neq 0$ then $c$ is a vertical asymptote
- Find the roots: The point(s) $x = c$ is a root if $p(c) = 0$ AND $q(c) \neq 0$
- Find horizontal asymptotes (if any): Take the limit as $x$ tends to positive and negative infinity by looking at the leading terms of $p$ and $q$
- Sketch the asymptotes and roots
- Analyze the first and second derivatives to determine the shape
- Sketch using the asymptotes, critical points, IPs and intercepts

**Polynomials (domain is all real $x$ values)**

**Linear** $f(x) = mx + b$:
- $b$ is the $y$-intercept
- $m$ is the slope of the line (rise / run)

**Quadratic** $f(x) = ax^2 + bx + c$:
- The axis of symmetry is $-b / 2a$
- The discriminant $b^2 - 4ac$ gives the number of $x$-intercepts
- The sign of $a$ determines whether it opens up or down

**Other Polynomials**

$f(x) = a_nx^n + a_{n-1}x^{n-1} + ... + a_1x + a_0$

- The leading term corresponds to the highest power of $x$
- Using the leading term, evaluate the limit at positive and negative infinity
- If the polynomial factors, find the roots
- Analyze the first and second derivatives to determine the shape
- Sketch using the critical points, IPs, intercepts
2. **Transcendental**

Exponentials \( f(x) = k \cdot b^x \)

- \( b > 1 \) implies growth
- \( b < 1 \) implies decay
- If \( k > 0 \), the function will always be positive. As \( x \) tends to infinity, it will tend to 0 (for decay) and infinity (for growth)
- If \( k < 0 \), the function will always be negative (it is flipped upside down)
- The \( y \) intercept is \( k \)
- Shifts:
  \( f(x) = k\cdot b^x + c \), shift up (\( c > 0 \)) or down (\( c < 0 \)) by \( c \)
  \( f(x) = k\cdot b^{x+c} \), shift left (\( c < 0 \)) or right (\( c > 0 \)) by \( c \)

Logs \( f(x) = \ln[g(x)] \)

- This is log base \( e \)
- The domain is restricted to \( g(x) > 0 \)
- It has a vertical asymptote at \( g(x) = 0 \)
- \( \ln[g(x)] \) tends to infinity as \( g(x) \) tends to infinity

Trig Functions

Hyperbolic Trig Functions
How to Sketch the Graph of a Function $f(x)$:

**Analyze the First and Second Derivatives to Determine Shape**

1. Find $f'(x)$
2. Find critical points (CP) – wherever $f'(x) = 0$ or undefined
3. Find $f''(x)$
4. Find inflection points (IP) – wherever $f''(x) = 0$
5. Make sign diagram for $f'(x)$ and $f''(x)$ which contains all CP’s, IP’s (and vertical asymptotes, if there are any)
6. Below the sign diagram, sketch the “shape” of the graph (i.e. increasing “/”, decreasing “\”, horizontal “—” etc.)
7. Find the actual critical points by finding $f(CP), f(IP)$, etc.
8. Plot CP’s and IP’s, and $x$ and $y$ intercepts.
9. Sketch