Factoring Quadratics with Nonzero \( a \):  

Students usually struggle with finding factors of the quadratic \( ax^2 + bx + c \) with nonzero \( a \). Remember that one can always use the quadratic formula. But one could find the factors in very clever ways…

As Presented in Class:

By example, let’s look at \( 4x^2 -19x +12 \)

Step 1: Multiply \( a \) and \( c \).

\( 4(12) = 48 \)

Step 2: “Rewrite” the equation by removing the \( a \) and replacing the \( c \)

\( x^2 -19x +48 \)

Step 3: Factor as usual

The factors of 48 are 1 & 48, 2 & 24, 3 & 16, 4 &12, 6 & 8

The two factors when added together to give you 19 are 3 & 16

\( x^2 -19x +48 = (x-16)(x-3) \)

Step 4: Of the numbers just found, find the factorization equivalent to \( a \) and ‘shift’ to \( a \’s \) place

\( (x-16)(x-3) \)

16 has factors of 4 & 4. Shifting we get \( (x-4)(4x-3) \)

Let’s look at \( 2x^2 + x -6 \)

Step 1: Multiply \( a \) and \( c \).

\( 2(-6) = -12 \)

Step 2: “Rewrite” the equation by removing the \( a \) and replacing the \( c \)

\( x^2 + x -12 \)

Step 3: Factor as usual

The factors of 12 are 1 & 12, 2& 6, 3 & 4

The two factors when subtracted give you 1 are 3 & 4

\( x^2 + x -12 = (x+4)(x-3) \)

Step 4: Of the numbers just found, find the factorization equivalent to \( a \) and ‘shift’ to \( a \’s \) place

\( (x+4)(x-3) \)

4 has factors of 2 & 2. Shifting we get \( (x+2)(2x-3) \)
This process works well until you have a problem similar to the one below…

Let’s look at $18x^2 + 27x + 10$
Step 1: Multiply $a$ and $c$.
$18(10) = 180$
Step 2: “Rewrite” the equation by removing the $a$ and replacing the $c$
$x^2 + 27x + 180$
Step 3: Factor as usual
The factors of 180 are 1&180, 2&90, 3&60, 4&45, 5&36, 6&30, 9&20, 10&18, 12&15
The two factors when added give you 27 are 12 & 15
$x^2 + 27x + 180 = (x+12)(x+15)$
Step 4: The difference now is that one number is not prime
We need to look at the factors of 12 and 15 when multiplied give you 18 ($a$) and 10 ($c$), respectively
12 has factors of 1&12, 2&6, 3&4
15 has factors of 1&15, 3&5
Looking at 2&6, 3&5… $2(5) = 10$ and $3(6) = 18$
Shifting we get $(3x+2)(6x+5)$

I recommend that you find a way that works for you and stick with it.
Some others are presented below.

**Quadratic Formula:**

$18x^2 + 27x + 10$

$x = \frac{-27 \pm \sqrt{27^2 - 4(18)(10)}}{2(18)} = \frac{-27 \pm 3}{36} = \frac{-2 \pm 5}{6}$

$\left( x + \frac{2}{3} \right) \left( x + \frac{5}{6} \right)$

$(3x+2)(6x+5)$

Always cheap and easy.

**From Purple Math:**

To factor a “hard” quadratic (with three coefficients not equal to one) visit the website
http://www.purplemath.com/modules/factquad2.htm

**Using Algebra Tiles:**

For teaching using algebra tiles, visit
http://regentsprep.org/Regents/math/faceq/TRFacEq.htm