Assignment 5

Due: Thursday November 13

1. On the website there is a data set labeled HW5. Assume this data is from a random sample. A scientist wishes to test the following hypothesis for the population mean at the $\alpha = 0.05$ level:

$$H_0 : \mu = \mu_0$$
$$H_1 : \mu \neq \mu_0$$

   The scientist wishes to test whether her colleagues are correct in believing that $\mu_0 = 4$. She does not agree with the general convention that this is the case.

   (a) Determine the appropriate observed test statistic and calculate the critical value under the null hypothesis. Then decide if you reject or fail to reject the null hypothesis.

   (b) Determine the p-value for the test and decide if you reject or fail to reject the null hypothesis.

   (c) Construct a 95% confidence interval. Based on the confidence interval, decide if you reject or fail to reject the null hypothesis.

2. Please do the following exercises from 'Applied Multivariate Statistical Analysis' by Johnson and Wichern. You can use R for whatever question you see fit. **Make sure to show details.**

   (a) 5.1

   (b) 5.2

   (c) 5.3 (except for Wilk's Lambda)

   (d) 5.4

   (e) 5.9

   (f) 5.10

   (g) 5.11

   (h) 6.1

3. **Extra Credit** (these can be turned in any time before the final exam):

(a) Let $y_1, \ldots, y_n \overset{\text{iid}}{\sim}$ exponential (λ). Thus $f(y_i|\lambda) = (\lambda)exp(-\lambda y_i)$, $0 \leq y_i < \infty$ and $\lambda > 0$. The $E(y_i) = 1/\lambda$ and the $V(y_i) = 1/\lambda^2$.

i. Find the method of moments estimator of λ.

ii. Find the maximum likelihood estimator estimator of λ. Make sure you actually have a maximum and not a minimum.

iii. Suppose, our prior belief about λ is described by a gamma distribution with parameters α, β, thus $\lambda \sim$ gamma(α, β). This is the conjugate prior for λ. Determine the posterior distribution of $\lambda|y_1, \ldots, y_n$.

(b) Suppose we have data $\boldsymbol{y} = (\boldsymbol{y_1}, \boldsymbol{y_2}, \ldots, \boldsymbol{y_n})^T \overset{\text{iid}}{\sim} MVN_p(\boldsymbol{\mu}, \Sigma)$. Thus, for each $\boldsymbol{y_i}$ the density of the distribution is given as:

$$f(\boldsymbol{y_i}) = (\sqrt{2\pi})^{-p}|\Sigma|^{-1/2}exp(-\frac{1}{2}(\boldsymbol{y_i} - \boldsymbol{\mu})^T\Sigma^{-1}(\boldsymbol{y_i} - \boldsymbol{\mu})).$$

Also, assume that Σ is known and thus is not random. Now our prior belief about $\boldsymbol{\mu}$ is described by a multivariate normal distribution with mean vector $\boldsymbol{\gamma}$ and covariance Ω, thus $\boldsymbol{\mu} \sim MVN_p(\boldsymbol{\gamma}, \Omega)$. This is the conjugate prior for $\boldsymbol{\mu}$. Determine the posterior distribution of $\boldsymbol{\mu}|\boldsymbol{y_1}, \ldots, \boldsymbol{y_n}$. Be thorough with your details.