Assignment 6

Due: Thursday May 1

[20 points]

1. Runs your respective experiments and send me the data via e-mail. Also write one page (typed) clearly outlining how the data was collected. From this page I should have no trouble understanding the data.

2. (from Peter Hoff) Means and Variances: Consider an unbalanced two-factor experiment with the following table of means and sample sizes:

	$F_2 = 1$	$F_2 = 2$
$F_1 = 1$	$\bar{y}_{11.}, r_{11}$	$\bar{y}_{12.}, r_{12}$
$F_1 = 2$	$\bar{y}_{21.}, r_{21}$	$\bar{y}_{22.}, r_{22}$

   Let

   - μ_{ij} be the true sample mean in cell (i, j)
   - $\mu_{i.} = (\mu_{i1} + \mu_{i2})/2$
   - $\mu_{.j} = (\mu_{1j} + \mu_{2j})/2$
   - $\mu_{..} = (\mu_{11} + \mu_{12} + \mu_{21} + \mu_{22})/4$
   - $V(y_{ijk}) = \sigma^2$

   (a) Let $\bar{y}_{i..}$ be the mean of all observations with $F1 = i$ (the standard marginal mean) and let $\hat{y}_{i..} = (\bar{y}_{11.} + \bar{y}_{12.})/2$ (the Least Squares mean).

      i. What is the $E(\bar{y}_{i..})$?
      ii. What is the $V(\bar{y}_{i..})$?
      iii. What is the $E(\hat{y}_{i..})$?
      iv. What is the $V(\hat{y}_{i..})$?

3. (Required for Statistics Students — extra credit for others): Consider the following one-factor model:

$$y_{i,j} = \mu + a_i + \epsilon_{i,j}; \qquad \epsilon_{i,j} \overset{\text{iid}}{\sim} \text{normal}(0, \text{variance} = \sigma^2), \text{ for } i = \{1, \ldots, t_a\} \text{ and } j = \{1, \ldots, r\}.$$

   Show that the $E(MSA) = \sigma^2 + \frac{r \sum_{i=1}^{t_a} (a_i)^2}{t_a - 1}$.