

Assignment 5

Due: Thursday April 17

[20 points]

1. Consider a one factor completely randomized design. For each $i = 1, \dots, t = 3$, let $Y_{i,1}, \dots, Y_{i,r}$ be independent samples from a distribution with mean μ_i and variance σ^2 . Also for known constants $k = (1, -1/2, -1/2)$ find the following quantities:
 - (a) $E(\bar{y}_i)$.
 - (b) $Var(\bar{y}_i)$.
 - (c) $E(\hat{C})$, where $\hat{C} = \sum_{i=1}^3 k_i \bar{y}_i$.
 - (d) $Var(\hat{C})$.
2. Consider a two factor completely randomized design. For each $i = 1, \dots, t_1 = 2$ and $j = 1, \dots, t_2 = 2$ let $Y_{i,j,1}, \dots, Y_{i,j,r}$ be independent samples from a distribution with mean $\mu_{i,j}$ and variance σ^2 . Determine the following quantities:
 - (a) $E(\bar{y}_{ij})$.
 - (b) $Var(\bar{y}_{ij})$.
 - (c) Provide an estimator for $C = (\mu_{1,1} - \mu_{2,1}) - (\mu_{1,2} - \mu_{2,2})$.
 - (d) $E(\hat{C})$.
 - (e) $Var(\hat{C})$.
3. (Montgomery) An experiment is conducted to study the influence of three types of face-plate glass and operating temperature in the light output of an oscilloscope. The data are on the website (osci.txt).
 - (a) Find an appropriate model for the data at the $\alpha = 0.05$ level.
 - (b) Write out the treatment effects model symbolically — clearly present the assumptions.
 - (c) Make sure to check these assumptions through residual analysis.
 - (d) What is the LSD for the glass factor? Rank the levels of the factor.
 - (e) What is the LSD for the temperature factor? Rank the levels of the factor.
 - (f) Provide a 95% confidence interval for $(\mu_{G1,T100} - \mu_{G3,T100}) - (\mu_{G1,T150} - \mu_{G3,T150})$. G1 and G3 are glass-types one and three. T100 and T150 are temperatures 100 and 150.

4. (Montgomery) Consider a three-factor model:

$$\begin{aligned}y_{i,j,k} &= \mu + a_i + b_j + c_k + (ab)_{i,j} + (bc)_{j,k} + (ac)_{i,k} + \epsilon_{i,j,k}; \\i &= 1, \dots, t_a, \\j &= 1, \dots, t_b, \\k &= 1, \dots, t_c.\end{aligned}$$

- Notice that there is only one replicate. Write down the analysis of variance table. What would you use as the MSE to test the hypotheses?

Project Get together in your groups. Among the possibilities you discussed before the midterm, choose three factors for your experiments and only have two levels for each factor. How many replicates will you have? Conduct the experiment. Remember to record any additional information which may be relevant (i.e. uncontrollable but measurable variables).