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ELLIPTIC H^2-VOLTERRA PROJECTION AND THE H^1-GALERKIN METHODS FOR THE INTEGRO-DIFFERENTIAL EQUATIONS OF EVOLUTION

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Abstract. This paper studies H^1-Galerkin methods for the integro-differential equations of evolution. The elliptic H^2-Volterra projection is induced and then used in the derivations of error estimates for semi-discrete and full-discrete H^1-Galerkin methods. The optimal L^2, H^1 and H^2 norm error estimates are obtained.

1. Introduction

Consider the parabolic integro-differential equation

$$u_t = \nabla \cdot (a(x) \nabla u + \int_0^t b(x,t,\tau) \nabla u(x,\tau)d\tau) + f(u), \quad (x,t) \in \Omega \times (0,T], \quad (1.1a)$$

$$u(x,0) = u_0(x), \qquad x \in \Omega, \quad (1.1b)$$

$$u(x,t) = 0, \qquad (x,t) \in \partial\Omega \times (0,T], \quad (1.1c)$$

and the hyperbolic integro-differential equation

$$u_{tt} = \nabla \cdot (a(x) \nabla u + \int_0^t b(x,t,\tau) \nabla u(x,\tau)d\tau) + f(u), \quad (x,t) \in \Omega \times (0,T], \quad (1.2a)$$

$$u(x,0) = u_0(x), \ u_t(x,0) = v_0(x), \qquad x \in \Omega, \quad (1.2b)$$

$$u(x,t) = 0, \qquad (x,t) \in \partial\Omega \times (0,T], \quad (1.2c)$$

where $T > 0$ and $\Omega \subset R^n (n = 1,2,3)$ is a bounded domain with smooth boundary $\partial\Omega$.

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