1997,17(3):319–329

Mathematica Acta Scientia
数学物理学报

MIXED FINITE ELEMENTS FOR PARABOLIC INTEGRO-DIFFERENTIAL EQUATIONS [1]

Sun Pengtao (孙澎涛)
Institute of Mathematics, Academia Sinica, Beijing 100080, China

Abstract In this paper, we study mixed finite elements for parabolic integro-differential equations, and introduce a kind of nonclassical mixed projection, its optimal L^2 and h^{-s} estimates are obtained. We define semi-discrete and full-discrete mixed finite elements for the equations, and obtain the optimal L^2 error estimates.

Key words Integro-differential Equations, Mixed Finite Element, Error Estimates

1 Introduction

We consider the following nonlinear parabolic integro-differential equations:

$$\begin{aligned}&(a)\ u_t = \nabla\cdot\{a(u)(\nabla u + \textstyle\int_0^t \nabla u(x,\tau)d\tau)\} + f(u), (x,t)\in\Omega\times(0,T],\\ &(b)\ u(x,0) = u_0(x), x\in\Omega,\\ &(c)\ u(x,t) = 0, (x,t)\in\partial\Omega\times(0,T].\end{aligned} \tag{1.1}$$

where $\Omega\subset R^2$ is a bounded domain with smooth boundary $\partial\Omega, T>0, a, u_0$ and f are known functions.

Problem (1.1) can arise from many physical processes such as some gas diffusion problems, heat transfer problems with memory, and etc. (see [1]).

For approximating the solution u, the classical finite element methods have been considered by several authors [2],[3],[4],[5] in recent years, and the author [6],[7],[8],[9] has studied classical and nonclassical finite element methods for integro-differential equations of evolution. But the mixed finite elements for problem (1.1) have not been considered by any authors. In this paper, we introduce a nonclassical mixed projection: Volterra-type mixed projection from which optimal L^2 error estimates can be derived for semi-discrete and full-discrete mxed finite elements for problem (1.1).

This paper is organized in the following way. In §2 we give mixed finite element formulations and some necessary preparations. The Volterra-type mxed projection will be introduced and studied in §3. Section 4 and 5 contain the error estimates for the semi-discrete and full-discrete mixed finite element aproximations, respectively for problem (1.1).

[1] Received Mar.5,1995; revised Jan.2,1996.