Problem 1 (10 Points)
1. Prove that any set of outer measure zero is measurable.
2. Prove that there are disjoint sets of real numbers $A$ and $B$ for which $m^*(A \cup B) < m^*(A) + m^*(B)$.

Problem 2 (10 Points)
1. Prove that the Cantor-Lebesgue function is not absolutely continuous.
2. Prove the existence of a Lebesgue measurable set which is not Borel.

Problem 3 (10 Points) Show that if $f$ is a bounded function on $E$ that belongs to $L^{p_1}(E)$, then it belongs to $L^{p_2}(E)$ for $p_2 > p_1$.

Problem 4 (10 Points) Is the space $L^\infty$ separable? Prove your answer.

Problem 5 (10 Points) Show that strong $L^p$ convergence implies weak $L^p$ convergence, and also that the (weak) limit of a weak convergent sequence is unique.

Problem 6 (10 Points) Let $E$ be the set of finite measure and $\delta > 0$. Then show that $E$ is the disjoint union of a finite collection of sets, each of which has measure less than $\delta$. 