**Problem 1** (10 Points) Derive the formula for the electric potential of an electric dipole at a distance $R$ with the position vector $a_R$ making an angle $\theta$ with the dipole vector $p$.

**Problem 2** (10 Points) Starting from static Maxwell’s equations, derive the following formula for the magnetic potential
\[
A = \frac{\mu_0}{4\pi} \int_C \frac{J}{R} \, dv
\]  
(1)

**Problem 3** (10 Points) Derive the Biot Savart law
\[
B = \frac{\mu_0}{4\pi} \oint_C \frac{dl \times a_R}{R^2}
\]  
(2)
starting from the expression for the magnetic potential in Equation (1).

**Problem 4** (10 Points) Starting from dynamic Maxwell’s equations, derive the following formula for the Electric field in terms of the electric and magnetic potentials.
\[
E = -\nabla V - \frac{\partial A}{\partial t}
\]  
(3)

**Problem 5** (10 Points) Using the formula for the electric potential in Equation (3) derive the following partial differential equation for the magnetic potential, and also state the formula for the retarded potential (solution to this equation), and also the phasor version of the same.
\[
\nabla^2 A - \mu \epsilon \frac{\partial^2 A}{\partial t^2} = -\mu J
\]  
(4)

**Problem 6** (20 Points) (a) Starting with the formula for the phasor retarded magnetic potential for a Hertzian dipole antenna given by
\[
A = a_z \frac{\mu_0 I dl}{4\pi} \left( e^{-j\beta R} \right)
\]  
(5)
(b) Using spherical coordinates, write the expressions for $A_R$, $A_\theta$ and $A_\phi$.
(c) Find the magnetic potential by performing the following operation
\[
H = \frac{1}{\mu_0} \nabla \times A = a_\phi \frac{1}{\mu_0 R} \left[ \frac{\partial}{\partial R} (RA_\theta) - \frac{\partial A_R}{\partial \theta} \right]
\]  
(6)
(d) Derive the formula for the near field magnetic field, and show the formula for the far field electric and magnetic field.
(e) Plot the E-plane and H-plane radiation pattern of the Hertzian dipole antenna.