Problem 1 (20 Points) The following is given: $\Omega = \{\omega_1, \omega_2, \omega_3\}$, $A = \{\omega_1, \omega_2\}$, and $B = \{\omega_2, \omega_3\}$.

1. Build some specific probability function on the discrete ($2^\Omega$) sigma algebra on this space.
2. What is $\nu_A$, the smallest sigma algebra containing $A$?
3. What is the sigma algebra generated by the random variable $\chi_A(\omega)$?
4. What is the random variable $E(\chi_A(\omega)/\nu_A)$?

Problem 2 (10 Points) Given $E(X^2) = 5$, provide a bound on $P(|X| \geq 10)$? Derive a proof for your bound that you provide.

Problem 3 (10 Points) Given $P(A_i) = (1/2)^i$, for $i = 1, 2, \cdots$, what is $P(\bigcap_{n=1}^{\infty} \bigcup_{i=n}^{\infty} A_i)$? Derive a proof for your answer.

Problem 4 (15 Points) We are given a sequence $X_i$ of random variables with binary distribution $P(X_i = 1) = P(X_i = -1) = 0.5$. Define $S_n = (\sum_{i=1}^{n} X_n)/n$.

1. Find, for any given $\epsilon > 0$, $\lim_{n \to \infty} P(|S_n| > \epsilon)$. Provide a proof of your result.
2. What is $P(\lim_{n \to \infty} S_n = 0)$? Give the name of the statement that gives this answer.
3. Find the variance $\sigma$ of $X_i$, and find out $\lim_{n \to \infty} P(-1 \leq \frac{S_n}{\sigma/\sqrt{n}} \leq 1)$. 

Dr. Pushkin Kachroo: pushkin@unlv.edu  http://faculty.unlv.edu/pushkin