Problem 1 (10 Points) Consider a non-deterministic push down automata (npda) with

\[ Q = \{ q_0, q_1, q_2, q_3 \} \text{ (internal control states)} \]
\[ \Sigma = \{ a, b \} \text{ (input alphabet)} \]
\[ \Gamma = \{ 0, 1 \} \text{ (stack alphabet)} \]
\[ z = 0 \text{ (stack start symbol)} \]
\[ F = \{ q_3 \} \text{ (set of final states)} \]

with initial state \( q_0 \), and the transition function \( \delta : Q \times (\Sigma \cup \{ \lambda \}) \times \Gamma \rightarrow \text{ finite subsets of } Q \times \Gamma^* \) given by

\[ \delta(q_0, a, 0) = \{ (q_1, 10), (q_3, \lambda) \} \]
\[ \delta(q_0, \lambda, 0) = \{ (q_3, \lambda) \} \]
\[ \delta(q_1, a, 1) = \{ (q_1, 11) \} \]
\[ \delta(q_1, b, 1) = \{ (q_2, \lambda) \} \]
\[ \delta(q_2, b, 1) = \{ (q_2, \lambda) \} \]
\[ \delta(q_2, \lambda, 0) = \{ (q_3, \lambda) \} \]

Show the sequence of internal states the system goes through when the input string is \( aabb \). Will this string be accepted or rejected? What language does this npda accept?

Problem 2 (10 Points) Compare the cardinality of the set of all rational numbers (positive, zero, and negative) and the set of natural numbers. Show the proof of your answer.