Problem 1 (10 Points) Use dynamic programming to find the shortest path from node $A$ to destination node $H$. Please show the value function clearly.

![Figure 1: Shortest Path Problem on a Labelled Digraph](image)

Problem 2 (10 Points) Given $t_0$ as the initial time, $t_f$ as the final time, and the initial state $x(t_0)$, for the system

$$\dot{x} = a(x(t), u(t), t)$$

(1)

using the dynamic programming method, design an optimal control law that minimizes the cost

$$J = h(x(t_f), t_f) + \int_{t_0}^{t_f} g(x(\tau), u(\tau), \tau) d\tau$$

(2)

Problem 3 (10 Points) For the system

$$\dot{x} = x(t) + u(t)$$

(3)

design an optimal control law that minimizes the following cost for a large fixed final time $T$.

$$J = \frac{1}{4} x^2(T) + \int_{0}^{T} \frac{1}{4} u^2(\tau) d\tau$$

(4)