Problem 1 (10 Points) (a) Solve the following optimization problem using the simplex algorithm. Please show all tableaus clearly.

Maximize : \( z = x_1 + 9x_2 + x_3 \)
subject to : \( x_1 + 2x_2 + 3x_3 \leq 9 \)
\( 3x_1 + 2x_2 + 2x_3 \leq 15 \)
with : \( x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \)

(b) Solve the following optimization problem using the simplex algorithm. Please show all tableaus clearly.

Minimize : \( z = x_1 + 9x_2 + x_3 \)
subject to : \( x_1 + 2x_2 + 3x_3 \leq 9 \)
\( 3x_1 + 2x_2 + 2x_3 \leq 15 \)
with : \( x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \)

Problem 2 (10 Points) Determine the symmetric dual of the program:

Minimize : \( z = 5x_1 + 2x_2 + x_3 \)
subject to : \( 2x_1 + 3x_2 + x_3 \geq 20 \)
\( 6x_1 + 8x_2 + 5x_3 \geq 30 \)
\( 7x_1 + x_2 + 3x_3 \geq 40 \)
\( x_1 + 2x_2 + 4x_3 \geq 50 \)
with : \( x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \)

Problem 3 (10 Points) Describe the steps of the branch and bound algorithm for (Linear) integer programming.