Problem 1 (10 Points) Solve the following optimization problem by hand. Moreover, give a graphical solution also.

Minimize : \( z = 0.04(x_1 - 95)^2 + 0.02(x_2 - 125)^2 \)
subject to : \( 0.2x_1 + 0.2x_2 \leq 20 \)
\( 0.8x_1 + 0.3x_2 \leq 60 \)
with : \( x_1 \geq 0, x_2 \geq 0 \)

Problem 2 (10 Points) Put the following program in standard form and provide an initial basic feasible solution.

Minimize : \( z = 25x_1 + 30x_2 \)
subject to : \( 4x_1 + 7x_2 \geq 1 \)
\( 8x_1 + 5x_2 \geq 3 \)
\( 6x_1 + 9x_2 \geq -2 \)
with : \( x_1 \geq 0, x_2 \geq 0 \)

Problem 3 (10 Points) Prove that the objective function of the following system assumes its minimum at an extreme point of \( \mathcal{S} \), provided a minimum exists and that \( \mathcal{S} \) is bounded.

Minimize : \( z = C^T X \)
subject to : \( AX = B \)
with : \( X \geq 0 \)