**PROBLEM 1:** (10 points) Define the following terms:

1. What is a Cauchy sequence?
2. What condition does a limit of a sequence have to satisfy?

**PROBLEM 2:** (10 points) What is the limit of the following sequence of rational numbers?

\[
x_0 = 1 \\
x_{n+1} = \frac{1}{2} \left( x_n + \frac{2}{x_n} \right)
\]

What does this example prove about the completeness of the set of rational numbers?

**PROBLEM 3:** (10 points) Define the following terms:

1. What is a linear or a vector space?
2. What is a normed linear space?
3. What is a Banach space?

**PROBLEM 4:** (10 points) Define the following terms:

1. What is a metric space?
2. Can a metric be defined on any set such that the set becomes a metric space?

**PROBLEM 5:** (10 points) Give a precise definition of a dynamic system. How does the following model of differential inclusion satisfy the definition?

\[
\dot{x} \in f(t, x), \quad x(t) \in \mathbb{R}^n
\]

**PROBLEM 6:** (10 points) Consider the following standard linearized model of a frictionless pendulum.

\[
\ddot{\theta}(t) = -\theta(t)
\]

1. Find all the equilibrium points of this system.
2. Is the equilibrium point at origin an invariant set?
3. Give two examples of invariant sets for this system.
4. Is the equilibrium point at origin stable, uniformly stable, uniformly asymptotically stable, exponentially stable?

**PROBLEM 7:** (10 points) Consider the following standard linearized model of a pendulum with friction.

\[
\ddot{\theta}(t) = -2\dot{\theta} - \theta(t)
\]

1. Find all the equilibrium points of this system.
2. Is the equilibrium point at origin an invariant set?
3. Give two examples of invariant sets for this system.
4. Is the equilibrium point at origin stable, uniformly stable, uniformly asymptotically stable, exponentially stable?