Problem 1 (10 Points) Consider the harmonic oscillator

\[
\begin{align*}
\dot{x}_1(t) &= x_2 \\
\dot{x}_2(t) &= -x_1
\end{align*}
\]

Using Poincaré-Bendixson criterion, find out about the existence of periodic orbits.

Problem 2 (10 Points) Consider the system

\[
\begin{align*}
\dot{x}_1(t) &= x_2 \\
\dot{x}_2(t) &= ax_1 + bx_2 - x_1^2 - x_1^3
\end{align*}
\]

and let $D$ be the whole plane. Using Bendixson criterion, find out about the existence of periodic orbits.

Problem 3 (10 Points) Find all equilibrium points and determine the type of each isolated equilibrium point. Also, draw the approximate phase plane plots near the three equilibrium points.

\[
\begin{align*}
\dot{x}_1(t) &= 2x_1 - x_1x_2 \\
\dot{x}_2(t) &= 2x_1^2 - x_2
\end{align*}
\]

Problem 4 (10 Points) Solve the ODE $\dot{x} = -x^2$ with $x(0) = -1$. What happens to the solution at $t = 1$. Show the relevance of the local and global Lipschitz conditions for the well-posedness of this system.

Problem 5 (10 Points) Find the stability of the equilibrium point for the system $\dot{x} = -x^3$ using the Lyapunov method by using a quadratic Lyapunov function. Test for local and global stability and asymptotic stability by clearly showing which Lyapunov theorems are applicable and how.