PROBLEM 1: (10 points) Give a precise definition of vector spaces and give one example each of a finite dimensional and infinite dimensional vector space.

PROBLEM 2: (10 points) Give a precise definition of subspaces and give one example of each for the two vector space examples in PROBLEM 1.

PROBLEM 3: (10 points) Give a precise definition of normed vector spaces and give one example each of a finite dimensional and infinite dimensional normed vector space.

PROBLEM 4: (10 points) Give an example of an open subset and a closed subset from your two examples of vector spaces in PROBLEM 1.

PROBLEM 5: (10 points) Show that the space of continuous functions on $[0, 1]$ with the norm defined by $\|x\| = \int_0^1 |x(t)| dt$. Show that this space is not complete.

PROBLEM 6: (10 points) (a) Give an example of an upper-semicontinuous function. (a) Give an example of a lower-semicontinuous function. (a) Give an example of a compact set.

PROBLEM 7: (10 points) State the Weierstrass theorem, and give a specific example of a discontinuous function where this theorem can be applied.

PROBLEM 8: (10 points) (a) Give a precise definition of Hilbert space and give one example. (b) State the Projection Theorem.