PROBLEM 1: Find the value function for the given graph, and find the next state from each state following the shortest paths to the final state F. (10 points)

PROBLEM 2: Derive the Hamilton-Jacobi partial differential equation for an optimal control problem given below. (10 points)

\[
\min J = h(x(t_f), t_f) + \int_{t_0}^{t_f} g(x(\tau), u(\tau), \tau) d\tau
\]
Subject to: \(\dot{x}(t) = a(x(t), u(t), t)\)

PROBLEM 3: Find the control law that minimizes the given cost for the dynamic system shown below? The final time is specified. (10 points)

\[
\min J = \frac{1}{4} x^2(T) + \int_0^T \frac{1}{4} u^2(t) dt
\]
Subject to: \(\dot{x}(t) = x(t) + u(t)\)

PROBLEM 4: Find the control law that minimizes the given cost for the dynamic system shown below? The final time is specified. (10 points)

\[
\min J = \frac{1}{2} x^T(t_f) H x(t_f) + \int_{t_0}^{t_f} \frac{1}{2} x^T(t) Q(t) x(t) + u^T(t) R(t) u(t) dt
\]
Subject to: \(\dot{x}(t) = A(t) x(t) + B(t) u(t)\)