PROBLEM 1: Graphically solve the following linear programming problem. Show clearly the feasible region on the graph. (10 points)

Maximize: \( z = 120x_1 + 80x_2 \)
Subject to: \( 2x_1 + x_2 \leq 6; \ 7x_1 + 8x_2 \leq 28; \ x_1 \geq 0; \ x_2 \geq 0 \)

PROBLEM 2: Put the following program in the standard form. Identify the slack and surplus variables. Generate an initial feasible solution for the Simplex algorithm. (10 points)

Minimize: \( z = x_1 + 2x_2 + 3x_3 \)
Subject to: \( 3x_1 + 4x_3 \leq 5; \ 5x_1 + x_2 + 6x_3 = 7; \ 8x_1 + 9x_3 \geq 2; \ x_1 \geq 0; \ x_2 \geq 0 \)

PROBLEM 3: A container in the shape of a right circular cylinder with no top has surface area \( 3\pi \) m\(^2\). What height \( h \) and base radius \( r \) will maximize the volume of the cylinder? (5 points)

Maximize: \( z = f(x_1, x_2, x_3) \)
Subject to: \( g_1(x_1, x_2, x_3) = 0; \ g_2(x_1, x_2, x_3) = 0; \)

PROBLEM 4: Write the Lagrangian function for the following problem, and state the necessary condition for a maximum. (5 points)

Maximize: \( z = f(x_1, x_2, x_3) \)
Subject to: \( g_1(x_1, x_2, x_3) = 0; \ g_2(x_1, x_2, x_3) \leq 0; \)

PROBLEM 5: State the necessary Kuhn-Tucker conditions for a maximum. (5 points)

Maximize: \( z = f(x_1, x_2, x_3) \)
Subject to: \( g_1(x_1, x_2, x_3) = 0; \ g_2(x_1, x_2, x_3) \leq 0; \)

PROBLEM 6: Given an initial guess for a solution to the following unconstrained maximization problem, what will be the update to this initial guess using (a) the method of steepest ascent, and (b) Newton Raphson method? (5 points)

Maximize: \( z = f(x), \ x \in \mathbb{R}^n \)

PROBLEM 7 (Take home): (a) Minimize using Kuhn-Tucker conditions: \( z = x_1^2 + x_2^2 \); subject to: \( x_1 + x_2 - 1 \leq 0 \); and (b) Minimize using Kuhn-Tucker conditions: \( z = x_1^2 + x_2^2 \); subject to: \( x_1 + x_2 - 1 \geq 0 \); (10 points)