Problem 1: (a) Design a state feedback controller to stabilize the origin of system (1); (b) Design an output feedback controller to stabilize the origin of system (1). (10 points)

\begin{align*}
\dot{x}_1 &= x_1 + x_2 \\
\dot{x}_2 &= x_1 x_2^2 - x_1 + u \\
y &= -x_1^3 + x_2 \\
\end{align*} \hspace{1cm} (1)

Problem 2: Design a tracking control problem for system (2) such that the output tracks the signal $\sin(t)$. Make sure that you consider the zero dynamics of the system, if any exist. (10 points)

\begin{align*}
\dot{x}_1 &= -x_1 + \frac{2 + x_2^2}{1 + x_3^2} u \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= x_1 x_3 + u \\
y &= x_2 \\
\end{align*} \hspace{1cm} (2)

Problem 3: Is system (3) feedback linearizable (check both conditions)? If yes, design a feedback control law and a change of variables for feedback linearization. (Hint: Try an obvious function for $h(x)$, such as the one in the homework) (15 points)

\begin{align*}
\dot{x}_1 &= x_1 + x_2 \\
\dot{x}_2 &= 3x_1^2 x_2 + x_1 + u \\
\end{align*} \hspace{1cm} (3)