STEREO VISION (motion detection interpretation)

\[
\frac{p_x}{f} = -\frac{v + x}{z} \quad -1
\]
\[
\frac{p_y}{f} = \frac{v - x}{z} \quad -2
\]

1 + 2 \Rightarrow z (p_x - p_y) = 2hf

\[
z = \frac{2hf}{p_y - p_x}
\]

ABSOLUTE ORIENTATION

\[r_x = Rr_x + r_0\]

\[
l_{11}x_1 + l_{12}x_2 + l_{13}z_p + l_{14} = z_n \\
l_{12}x_2 + l_{22}x_2 + l_{23}z_p + l_{24} = z_n \\
l_{31}x_1 + l_{32}x_2 + l_{33}z_p + l_{34} = z_n
\]

Given \((x_1, y_1, z_1)\) and \((x_2, y_2, z_2)\)

find \(R\) and \(r_0\).

In general, use multiple points.

Define error for \(i\)th point \(e_i = (Rr_i + r_0) - r_{x,i}\)

Least square solution: parameters obtained by

\[\arg\min \sum_{i} e_i^2\]
Relative Orientation

Projection of \( r_e = (x_e, y_e, z_e) \) is

\[
x'_e = \frac{x_e f}{z_e} \quad \text{and} \quad y'_e = \frac{y_e f}{z_e}
\]

of \( r_e = (x_e, y_e, z_e) \) is

\[
x'_{e'} = \frac{x_{e'} f}{z_{e'}} \quad \text{and} \quad y'_{e'} = \frac{y_{e'} f}{z_{e'}}
\]

We need to find \( R \) and \( r_0 \) given \((x'_e, y'_e)\) and \((x'_{e'}, y'_{e'})\).

We have:

\[
\begin{align*}
x_{11} x'_e + x_{12} y'_e + x_{13} f + x_{14} h &= \frac{x_e z_e}{z_e} \\
x_{21} x'_e + x_{22} y'_e + x_{23} f + x_{24} h &= \frac{y_e z_e}{z_e} \\
x_{31} x'_e + x_{32} y'_e + x_{33} f + x_{34} h &= \frac{f z_e}{z_e}
\end{align*}
\]

- 3 equations + 14 unknowns \( x_{11}, \ldots, x_{24}, z_e, z_e \).
- Each additional point \( \rightarrow \) 3 more equations.
- \( x_{11}, x_{12}, x_{13}, x_{14}, x_{21}, x_{22}, x_{23}, x_{24}, x_{31}, x_{32}, x_{33}, x_{34}, f, z_e, z_e \) are the variables. If you have a solution for \( x_{11}, x_{12}, x_{13}, x_{14}, z_e, z_e \) you must multiply all of these by a constant \( k \) to get another solution. This shows solution is not unique. (The solution is invariant to scaling of all distances).
- To get a unique solution, we need some additional constraint. E.g. for \( r_0 = (k x'_e, k y'_e, k z'_e) \),

\[
x_0 \cdot x_0 = 1
\]
For left image
\[ x_l = x_l', s, \quad y_l = y_l', s, \quad z_l = f_s \]

In the right coordinate system
\[
\begin{align*}
    x_r &= (a_{11} x_l' + a_{12} y_l' + a_{13} b) s + c x_l \\
    y_r &= (a_{21} x_l' + a_{22} y_l' + a_{23} b) s + c y_l \\
    z_r &= (a_{31} x_l' + a_{32} y_l' + a_{33} b) s + c z_l
\end{align*}
\]

They project to
\[ x_l' = \frac{x_l f}{z_l}, \quad y_l' = \frac{y_l f}{z_l} \]

Rewrite (1) as
\[ x_r = as + u, \quad y_r = bs + v, \quad z_r = cs + w \]

Then
\[
\begin{align*}
    x_l' &= \frac{a}{c} + \frac{cu - aw}{c} \cdot \frac{1}{cs + w} \\
    y_l' &= \frac{b}{c} + \frac{cv - bw}{c} \cdot \frac{1}{cs + w}
\end{align*}
\]

For \( s = 0 \) gives \( \frac{x_l'}{b} = \frac{u}{w} \), \( \frac{y_l'}{b} = \frac{v}{w} \)

\( s \to \infty \) gives \( \frac{x_l'}{b} = \frac{a}{c} \), \( \frac{y_l'}{b} = \frac{1}{c} \)

\( \therefore (2) \) describes a straight line equation (epipolar line)
Computing Depth

Given \((x', y')\) and \((x', y')\) for some point on an object.

Equation

\[
\left( 211 \frac{x'}{f} + 212 \frac{y'}{f} + 213 \right) z_1 + 214 = \frac{x'}{f} z_1
\]

\[
\left( 221 \frac{x'}{f} + 222 \frac{y'}{f} + 223 \right) z_1 + 224 = \frac{y'}{f} z_1
\]

\[
\left( 231 \frac{x'}{f} + 232 \frac{y'}{f} + 233 \right) z_1 + 234 = z_1
\]

Use any two equations to compute \(z_1\) and \(z_2\), and then

\[
r_1 = (x', y', z')^T = \left(\frac{x'}{f}, \frac{y'}{f}, 1\right)^T z_1
\]

\[
r_2 = (x', y', z')^T = \left(\frac{x'}{f}, \frac{y'}{f}, 1\right)^T z_1
\]
Exterior Orientation

\[ \begin{align*}
    r_{11} x_a + r_{12} y_a + r_{13} z_a + r_{14} &= x_c \\
    r_{21} x_a + r_{22} y_a + r_{23} z_a + r_{24} &= y_c \\
    r_{31} x_a + r_{32} y_a + r_{33} z_a + r_{34} &= z_c
\end{align*} \]

From images,

\[ \begin{align*}
    \frac{x'}{b} &= \frac{x_c}{z_c}; \quad \frac{y'}{b} = \frac{y_c}{z_c}
\end{align*} \]

\[ \begin{align*}
    x' &= \frac{r_{11} x_a + r_{12} y_a + r_{13} z_a + r_{14}}{r_{31} x_a + r_{32} y_a + r_{33} z_a + r_{34}} \\
    y' &= \frac{r_{21} x_a + r_{22} y_a + r_{23} z_a + r_{24}}{r_{31} x_a + r_{32} y_a + r_{33} z_a + r_{34}}
\end{align*} \]  (E-2)

using least squares and given \((x_1, y_1), (x_2, y_2, z_2)\) for multiple camera attack parameters.

Interior Orientation

From camera to image (including all affine effects)

\[ \begin{align*}
    x' &= a_{11} \left( \frac{x_c}{z_c} \right) + a_{12} \left( \frac{y_c}{z_c} \right) + a_{13} \\
    y' &= a_{21} \left( \frac{x_c}{z_c} \right) + a_{22} \left( \frac{y_c}{z_c} \right) + a_{23}
\end{align*} \]  (I-1)

Combine I-1 with E-1 to get

\[ \begin{align*}
    \frac{x'}{b} &= \frac{s_{11} x_a + s_{12} y_a + s_{13} z_a + s_{14}}{s_{31} x_a + s_{32} y_a + s_{33} z_a + s_{34}} \\
    \frac{y'}{b} &= \frac{s_{21} x_a + s_{22} y_a + s_{23} z_a + s_{24}}{s_{31} x_a + s_{32} y_a + s_{33} z_a + s_{34}}
\end{align*} \]  (I-2)
Finding Conjugate Points (The Correspondence Problem)

- Analyze images and identify features separately
  - e.g., template matching
  - Find the location of maximum correlation.
  - Use the epipolar constraint.

Grey-Level Matching

\[
\begin{align*}
\frac{x'_p}{f} &= \frac{x + b/2}{2} \quad \frac{x'_o}{f} = \frac{x - 6b}{2} \\
\frac{y'_p}{f} &= \frac{y}{2} \\
x'_p - x'_o &= \frac{b}{2}
\end{align*}
\]

\[
\begin{align*}
\therefore \quad x &= b \frac{x'_p + x'_o}{x'_p - x'_o} \\
y &= b \frac{y'_p + y'_o}{x'_p - x'_o} \\
z &= b \frac{f}{x'_p - x'_o}
\end{align*}
\]

We want to find \( z(x'_i, y'_i) \) \( z = b f / (x'_i - x'_o) \) so that

\[
\begin{align*}
I(x'_i, y'_i) &= I(x'_o, y'_o) \\
I(b \frac{x + b/2}{z(x'_i, y'_i)}, y'_i) &= I(b \frac{x - 6b/2}{z(x'_o, y'_o)}, y'_i)
\end{align*}
\]

using \( \frac{x'_i}{f} = \frac{x}{2} \) and \( d(x'_i, y'_i) = \frac{b f}{z} \)
Find \( d(x',y') \) s.t.

\[
I_k \left( x' + \frac{1}{2} d(x',y') y' \right) = I_k \left( x' - \frac{1}{2} d(x',y') y' \right)
\]

We want \( \varepsilon \) and hence \( d \) to vary smoothly.

Find \( d \) that minimizes

\[
\int \left[ (E_1 - E_1)^2 + \lambda (\nabla^2 d)^2 \right] \, dx'dy'
\]

Calculate of Variation Problem: Solution obtained by Euler equation.

\[
\frac{\partial F_d}{\partial x'} - \frac{\partial^2 F_d}{\partial y' \partial x'} = 0
\]

\[
\Rightarrow \quad \nabla^2 (\nabla^2 d) = 2 (E_1 - E_1) \left( \frac{\partial E_1}{\partial x'} + \frac{\partial E_1}{\partial x''} \right)
\]

Iterative numerical schemes are available for this.

- Edge-based methods

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