Each problem is worth 10 points.

1. Find the minimizing control and the corresponding minimum cost for the system
   \[ \begin{align*}
   \dot{x}_1(t) &= 2x_1(t) \\
   \dot{x}_2(t) &= -3x_2(t) + u(t)
   \end{align*} \]
   
   \[ J(x(.), u(.)) = \int_0^\infty \{ x^T(t) \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} x(t) + u^2(t) \} dt \]

2. Assuming that the assumptions of the Kalman Filter are true, is the following true? Prove your answer.
   \[ E[\{x(t) - \hat{x}(t)\}x^T(t)] = E[\{x(t) - \hat{x}(t)\}\{x(t) - \hat{x}(t)\}^T] \]

3. Design a feedback controller that minimizes
   \[ J(x(.), u(.)) = E[25x^2(\infty) + u^2(\infty)] \]
   for the following system:
   \[ \begin{align*}
   \dot{x}(t) &= 5x(t) + 3u(t) + 2w(t) \\
   m(t) &= 3x(t)
   \end{align*} \]
   Given that \( S_w = 1 \)

4. Design an LQR based control design such that the steady state value for the output \( y(t) \) is 2 for the following system.
   \[ \begin{align*}
   \dot{x}(t) &= 3x(t) + u(t) \\
   y(t) &= 5x(t)
   \end{align*} \]
   You can take the value of any weights to be unity.