1. Rectangular Pulse

- The Fourier transform of \( x(t) \) is given in problem 1.13 on page 25 of the Schaum text:

\[
X(\omega) = 2\frac{\sin(a\omega)}{\omega}
\]

Another way of solving this problem is to recognize the rectangular pulse function as a sum of two unit step functions: \( x(t) = u(t+a) - u(t-a) \). Utilizing the addition and time-shifting properties of the Fourier transform, we can write the Fourier transform of \( x(t) \):

\[
X(\omega) = \left( \pi\delta(\omega) + \frac{1}{j\omega} \right)e^{j\omega a} - \left( \pi\delta(\omega) + \frac{1}{j\omega} \right)e^{-j\omega a}
\]

\[
= \left( \pi\delta(\omega) + \frac{1}{j\omega} \right)\left(e^{j\omega a} - e^{-j\omega a}\right)
\]

\[
= \left( \pi\delta(\omega) + \frac{1}{j\omega} \right) \cdot 2j \sin(\omega a)
\]

Since \( \sin(\omega a) = 0 \) at \( \omega = 0 \),

\[
= \frac{1}{j\omega} \cdot 2j \sin(\omega a) = 2\frac{\sin(a\omega)}{\omega}
\]

- The energy spectral density is given by \( ESD = |X(\omega)|^2 \), which in this case (since \( X(\omega) \) is real) becomes simply \( X^2(\omega) \), or

\[
ESD = 4\frac{\sin^2(a\omega)}{\omega^2}
\]

- Because \( x(t) \) is an energy signal and finite in duration, its power spectral density is equal to zero. See Lathi p.23 and Schaum pp.7-8.

2. Because the LTI system is an ideal low-pass filter (LPF) whose bandwidth \( W \) is greater than the bandwidth of the signal \( x(t) \), the output power of \( x(t) \), \( S_{x_o} \), is equal to the input power, \( S_{x_i} \). Given the power spectral density of the noise as \( k \) and the bandwidth of the LPF, we can calculate the output power of the noise signal by

\[
S_{n_o} = \frac{1}{2\pi} \int_{-W}^{W} k\partial\omega = \frac{kW}{\pi}
\]

Therefore, the signal-to-noise ratio of the system is given by

\[
SNR = \frac{S_{x_o}}{S_{n_o}} = \frac{S_{x_i} \cdot \pi}{kW}
\]

By inspection the bandwidth of the output signal is equal to \( W \) because the noise is equally distributed throughout all frequencies (as evidenced by its constant power spectral density of \( k \)), which is limited at the output by the LPF to \( W \).

3. Properties of the Dirac Delta distribution

- The solution to a) is found in the Schaum text in Problem 1.6a) on p.21
- The solution to b) is found in the Schaum text in Problem 1.7a) on p.21

4. This solution is found in the Schaum text in Problem 1.34 on pp.33-34

5. This solution is found in the Schaum text in Problem 1.43 on p.37