Problem 1 (20 Points)

1. What is the output of a Linear Time Invariant (LTI) system in terms of the input signal and the impulse response of the system? Derive this result for continuous as well as discrete time systems.

2. Prove that for continuous time LTI systems impulse response is the derivative of step response, and for discrete time, it is the difference of the step and shifted step response.

3. Prove that for continuous time LTI systems, complex exponential function are the eigenfunctions, and for discrete time, these are complex power functions. Derive the eigenvalues for both cases.

4. Prove that the derivative of the unit step function is the impulse for continuous time case. Derive the analogous result for the discrete time case.

Problem 2 (10 Points) The open-loop transfer function for an LTI system is given by

\[ G(s) = \frac{1}{(s - 1)} \]

1. Is the system stable?

2. Design a proportional feedback control law for this system. For what values of the gain \(k\) will the closed loop system be stable?