ECO 261-Fall 2009

Solutions for Problem Set 3

1) $\mu = \sum xP(x) = 1 * 0.07 + 2 * 0.19 + 3 * 0.28 + 4 * 0.30 + 5 * 0.16 = 3.29$

$\sigma^2 = \sum (x - \mu)^2 P(x) = (1 - 3.29)^2 * 0.07 + (2 - 3.29)^2 * 0.19 + ... + (5 - 3.29)^2 * 0.16 = 1.325$

$\sigma = \sqrt{1.325} = 1.151$

2) a)

X	$P(x)$	$F(x)$
0	0.10	0.10
1	0.08	0.18
2	0.07	0.25
3	0.15	0.40
4	0.12	0.52
5	0.08	0.60
6	0.10	0.70
7	0.12	0.82
8	0.08	0.90
9	0.10	1.00

b) $P(X \geq 5) = 0.08 + 0.10 + 0.12 + 0.08 + 0.10 = 0.48$

c) $P(3 \leq X \leq 7) = 0.15 + 0.12 + 0.08 + 0.10 + 0.12 = 0.57$

3) a)

X	$P(x)$	$F(x)$
0	0.10	0.10
1	0.15	0.25
2	0.19	0.44
3	0.26	0.70
4	0.19	0.89
5	0.11	1.00

b) $\mu = \sum xP(x) = 0 * 0.10 + 1 * 0.15..... + 5 * 0.11 = 2.62$

$\sigma^2 = \sum (x - \mu)^2 P(x) = (0 - 2.62)^2 * 0.10 + ... + (5 - 2.62)^2 * 0.11 = 2.093$

$\sigma = \sqrt{2.093} = 1.447$

4) a) $P(X \geq 1) = 1 - P(0)$

$$P(0) = \frac{5!}{0! * (5-0)!} * (0.25)^0(0.75)^5 = 0.237$$

$P(X \geq 1) = 1 - 0.237 = 0.763$

b) $P(X \geq 3) = 1 - P(X \leq 2) = 1 - [P(0) + P(1) + P(2)]$

$$P(0) = \frac{5!}{0! * (5-0)!} * (0.25)^0(0.75)^5 = 0.237$$
$$P(1) = \frac{5!}{1! * (5-1)!} * (0.25)^1(0.75)^4 = 0.395$$
$$P(2) = \frac{5!}{2! * (5-2)!} * (0.25)^2(0.75)^3 = 0.263$$

$P(X \geq 3) = 1 - P(X \leq 2) = 1 - [0.237 + 0.395 + 0.263] = 0.105$

5) a) $P(X = 5) = \frac{5!}{5! * (5-5)!} * (0.4)^5(0.6)^0 = 0.010$

b) $P(X \geq 3) = 1 - P(X < 3) = 1 - [P(0) + P(1) + P(2)]$

$$P(0) = \frac{5!}{0! * (5-0)!} * (0.4)^0(0.6)^5 = 0.077$$
$$P(1) = \frac{5!}{1! * (5-1)!} * (0.4)^1(0.6)^4 = 0.259$$
$$P(2) = \frac{5!}{2! * (5-2)!} * (0.4)^2(0.6)^3 = 0.345$$

$P(X \geq 3) = 1 - P(X < 3) = 1 - [0.077 + 0.259 + 0.345] = 0.319$

c) The majority out of four games (first game is already won) is equal to $P(X \geq 2)$

$P(X \geq 2) = 1 - P(X < 2) = 1 - [P(0) + P(1)]$

$$P(0) = \frac{4!}{0! * (4-0)!} * (0.4)^0(0.6)^4 = 0.129$$
$$P(1) = \frac{4!}{1! * (4-1)!} * (0.4)^1(0.6)^3 = 0.345$$

$P(X \geq 2) = 1 - P(X < 2) = 1 - [0.129 + 0.345] = 0.526$

d) The mean of the Binomial distribution is equal to its expected value.

$\Longrightarrow E(X) = \mu = n * P = 5 * 0.4 = 2$ The number of games Cubs is expected to win before the series.

6) The mean and the standard deviation of a Binomial distribution are as follows

$\mu = n * P = 2,000 * 0.032 = 64$

$\sigma = \sqrt{n * P * (1 - P)} = \sqrt{2,000 * 0.032 * 0.968} = 7.871$

b) Each failure costs \$10

$\Longrightarrow 10 * \mu = 10 * n * P = 10 * 2,000 * 0.032 = \$640$

$\Longrightarrow 10 * \sigma = 10 * \sqrt{n * P * (1 - P)} = 10 * \sqrt{2,000 * 0.032 * 0.968} = 78.71$

7) a) Because of the time dimension, this is a Poisson distribution.

$P(X < 2) = P(0) + P(1)$

$$P(0) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(3.2)^0 * e^{-3.2}}{0!} = 0.040$$
$$P(1) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(3.2)^1 * e^{-3.2}}{1!} = 0.130$$

$P(X < 2) = P(0) + P(1) = 0.170$

b) $P(X > 4) = 1 - P(X \leq 4) = 1 - [P(0) + P(1) + P(2) + P(3) + P(4)]$

$$P(0) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(3.2)^0 * e^{-3.2}}{0!} = 0.040$$
$$P(1) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(3.2)^1 * e^{-3.2}}{1!} = 0.130$$
$$P(2) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(3.2)^2 * e^{-3.2}}{2!} = 0.208$$
$$P(3) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(3.2)^3 * e^{-3.2}}{3!} = 0.222$$

$$P(4) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(3.2)^4 * e^{-3.2}}{4!} = 0.178$$

$P(X > 4) = 1 - P(X \leq 4) = 1 - [0.040 + 0.130 + 0.208 + 0.222 + 0.178] = 0.222$

8) $P(X \geq 3) = 1 - P(X \leq 2) = 1 - [P(0) + P(1) + P(2)]$

$$P(0) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(4.2)^0 * e^{-4.2}}{0!} = 0.014$$
$$P(1) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(4.2)^1 * e^{-4.2}}{1!} = 0.058$$
$$P(2) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(4.2)^2 * e^{-4.2}}{2!} = 0.123$$

$P(X \geq 3) = 1 - P(X \leq 2) = 1 - [0.014 + 0.058 + 0.123] = 0.805$

9) a) A random variable is a variable that takes on numerical values determined by the outcome of a random experiment.

b) The probability distribution function expresses the probability that the random variable (X) takes the value x (numerical value).

c) The cumulative probability function is the accumulation of the probability values that does not exceed any specified value x_0.