

ECO 261-Fall 2009

Solutions for Problem Set 2

1) a) The complement of event A is that it will take 4 days or less before the machinery becomes operational.

$$\bar{A} = \{1, 2, 3, 4\}$$

b) The intersection of A and B will be the event that it takes 5 days before the machinery becomes operational.

$$A \cap B = \{5\}$$

c) $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$

d) A and B are not mutually exclusive because $P(A \cap B) \neq 0$

e) $A \cap B = \{5\}$ $\bar{A} \cap B = \{1, 2, 3, 4\} \implies (A \cap B) \cup (\bar{A} \cap B) = \{1, 2, 3, 4, 5\} = B$

f) $A = \{5, 6, 7\}$ $\bar{A} \cap B = \{1, 2, 3, 4\} \implies A \cup (\bar{A} \cap B) = \{1, 2, 3, 4, 5, 6, 7\} = A \cup B$

2) a) The total number of outcomes in the sample space is $C_4^{16} = \frac{16!}{4!(16-4)!} = 1,820$

The number of ways to select 2 A's from the 10 available is $C_2^{10} = \frac{10!}{2!(10-2)!} = 45$

The number of ways to select 2 B's from the 6 available is $C_2^6 = \frac{6!}{2!(6-2)!} = 15$

The number of outcomes that satisfy the conditions 2 A's and 2 B's is $45 \cdot 15 = 675$. Therefore, the probability that

a randomly selected set of 4 will include 2 A's and 2 B's is $P = \frac{C_2^{10} * C_2^6}{C_4^{16}} = \frac{45 * 15}{1,820} = 0.3709$

b) The total number of outcomes in the sample space is $C_3^{10} = \frac{10!}{3!(10-3)!} = 120$

The number of ways to select 1 A from the 6 available is $C_1^6 = \frac{6!}{1!(6-1)!} = 6$

The number of ways to select 2 B's from the 4 available is $C_2^4 = \frac{4!}{2!(4-2)!} = 6$

The number of outcomes that satisfy the conditions 1 A and 2 B's is $6 \cdot 6 = 36$. Therefore, the probability that a

randomly selected set of 3 will include 1 A and 2 B's is $P = \frac{C_1^6 * C_2^4}{C_3^{10}} = \frac{6 * 6}{120} = 0.30$

3) a) $P(X < 3) = 0.29 + 0.36 + 0.22 = 0.87$

b) $P(X > 1) = 0.22 + 0.10 + 0.03 = 0.35$

c) By the first rule of probability theory, the sum of the probabilities of all outcomes in the sample space must sum to one.

4) a) $P(A) = 0.39 + 0.23 + 0.15 + 0.06 + 0.03 = 0.86$

b) $P(B) = 0.14 + 0.39 + 0.23 + 0.15 = 0.91$

c) $P(\bar{A}) = 1 - 0.86 = 0.14$

d) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.86 + 0.91 - 0.77 = 1.00$

e) $P(A \cap B) = 0.39 + 0.23 + 0.15 = 0.77$

f) Since $P(A \cap B) \neq 0$, the two events are not mutually exclusive.

5) a) $P(A) = 0.40$ $P(B) = 0.45$ and $P(A \cup B) = 0.85$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \implies 0.85 = 0.40 + 0.45 - P(A \cap B) \implies P(A \cap B) = 0$$

b) $P(A) = 0.60$ $P(B) = 0.45$ $P(A \cap B) = 0.30$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \implies P(A \cup B) = 0.60 + 0.45 - 0.30 = 0.75$$

c) $P(A) = 0.80$ $P(B) = 0.10$ $P(A \cap B) = 0.08$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.08}{0.10} = 0.80. \text{ A and B are independent events since } P(A \cap B) = P(A)P(B)$$

d) $P(A) = 0.30$ $P(B) = 0.40$ $P(A \cap B) = 0.30$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.30}{0.40} = 0.75. \text{ A and B are not independent events since } P(A \cap B) \neq P(A)P(B)$$

6) If A is the event "no graduate student is selected" then the number of combinations of 3 from 6 is $C_3^6 = \frac{6!}{3!(6-3)!} = 20$.

The number of combinations of 3 objects selected from 4 undergraduates is $C_3^4 = \frac{4!}{3!(4-3)!} = 4$. In other words, the committee will consist of only undergraduate students. Then, the probability of no graduate students for the committee is $P(\cdot) = 4/20 = 1/5$.

7) Let A-customer ask for assistance, B- customer makes a purchase, $A \cap B$ -both. Then,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \implies 0.30 + 0.20 - 0.15 = 0.35$$