ECO 441 Fall 2009

Solutions-1

1) a) The predicted average test score is

$$TestScore = 520.4 - 5.82 * 22 = 392.36$$

b) The predicted change in the classroom average test score is

$$\Delta TestScore = (520.4 - 5.82 * 19) - (520.4 - 5.82 * 23) = 23.28$$

c) Using the formula for $\hat{\beta}_0$ in equation (4.8), we know the sample average of the test scores across the 100 classroom is

$$\overline{TestScore} = \hat{\beta}_0 + \hat{\beta}_1 * \overline{CS} = 520.4 - 5.82 * 21.4 = 395.85$$

d) Use the formula for the standard error of the regression (SER) in equation (4.19) to get the sum of squared residuals:

$$SSR = (n-2)SER^2 = (100-2) * 11.5^2 = 12961$$

Use the formula for R^2 in equation (4.16) to get the total sum of squares:

$$TSS = \frac{SSR}{1 - 0.08} = 14087.5$$

The sample variance is $s_Y^2 = \dfrac{TSS}{n-1} = \dfrac{14087.5}{99} = 142.29$ Thus, standard deviation is $s_Y = \sqrt{s_Y^2} = 11.92$

2) a) Substituting $Height = 70, 65\ and\ 74$ inches into the equation, the predicted weights are $176.39, 156.69\ and\ 192.15$ pounds.

b)

$$\Delta Weight = 3.94 * \Delta Height = 3.94 * 1.5 = 5.91$$

c) We have the following relations: $1inch = 2.54cm\ and\ 1lb = 0.4536kg$. Suppose the regression equation in the centimeter-kilogram space is

$$Weight_i = \hat{\gamma}_0 + \hat{\gamma}_1 Height_i$$

The coefficients are $\hat{\gamma}_0 = -99.41 * 0.4536 = -45.092kg$, $\hat{\gamma}_1 = 3.94 * \frac{0.4536}{2.54} = 0.7036 kg\ per\ cm$. The R^2 is unit free, so it remains at $R^2 = 0.81$. The standard error of the regression is $SER = 1.02 * 0.4536 = 4.6267$.

3) a) The coefficient 9.6 shows the unit effect of Age on AWE, that is AWE is expected to increase by \$9.6 for each additional year of age. 696.7 is the intercept and usually has no meaning.

b) SER is in the same units as the dependent variable. Thus, SER is measures in dollars per week.

c) R^2 is unit free

d)

$$\begin{aligned} 696.7 + 9.6 * 25 &= 936.7 \\ 696.7 + 9.6 * 45 &= 1128.7 \end{aligned}$$

e) No. The oldest worker in the sample is 65 years old. 99 years is far outside the range of the sample data.

f) It depends on the sample size. if the sample size is greater than 30, then the distribution of the error terms can be approximated with a normal distribution due to Central Limit Theorem. Under the assumption that the sample size is less than 30, the age group reflects only college graduates who are likely earn more than average and therefore, the distribution may be right skewed.

g) $\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$ so that $\bar{Y} = \hat{\beta}_0 + \hat{\beta}_1 \bar{X}$ Thus, the sample mean of AWE is

$$696.7 + 9.6 * 41.6 = 1096.06$$

4) a) With $\hat{\beta}_1 = 0, \hat{\beta}_0 = \bar{Y}$ and $\hat{Y}_i = \hat{\beta}_0 = \bar{Y}$. Thus $ESS = 0$ and $R^2 = 0$

b) If $R^2 = 0$ then $ESS = 0$ so that $\hat{Y}_i = \bar{Y}$, which implies that $\hat{\beta}_1 = 0$ or that X_i is constant for all i. if X_i is constant for all i then $\sum_{i=1}^{n} (X_i - \bar{X})^2 = 0$ and $\hat{\beta}_1$ is undefined (see equation 4.7).

E.1) a)

$$AHE = -1.133 + 0.591 Age$$

Earnings increase, on average, by 0.591 dollars per hour when workers age by 1.

b) $Bob's Predicted Earning = -1.133 + 0.591 * 26 = 14.233$

$Alex's Predicted Earning = -1.133 + 0.591 * 30 = 16.597$

c) The R^2 is 0.04. This means that age explains a small fraction of the variability in earnings across individuals.

E.3) a)

$$Ed = 13.962 - 0.056 * Dist$$

The regression predicts that if colleges are built 10 miles closer to where students go to high school, average years of college will increase by 0.056 years.

b) $Bob\prime s \Pr edictedEd = 13.962 - 0.056 * 2 = 13.85$

$Bob\prime s \Pr edictedEd = 13.962 - 0.056 * 1 = 13.906$

c) The regression R^2 is 0.055, so that the distance explains only a very small fraction of years of completed education.

d) The SER is 1.786 years.