

A Hypothetical Bayesian Analysis Problem
(Modeled after a problem assigned by Prof. Eric van Marck of Princeton University)

Purpose:

This problem explores a simple application of Bayes' theorem. Part of the solution may seem counterintuitive...which is the point.

Background:

Read pages 128 – 142 of *Should We Risk It?* and review your notes and the handout from the Bayesian analysis lecture.

There has been a fatal car accident. Investigators think that there are only two possible causes:

$A = \{\text{the brakes failed}\}$

$A' = \{\text{the driver failed to apply the brakes}\}$

Investigators estimate the (prior) probability that event A' occurred at one in ten (or 0.1), before examining the accident site. Examination of wreckage will result in one of these events:

$B = \{\text{indication of brake failure}\}$

$B' = \{\text{no such indication}\}$

The investigators also know that they are not perfect, and suggest that

- The probability that they will recognize a brake failure if it happens is 0.8, and
- The probability that they will find no evidence of a brake failure if it did not happen is 0.9, or

$$P[B|A] = 0.8$$

$$P[B'|A'] = 0.9$$

Of course, the investigators never will know the truth, but they would like to understand what happened as accurately as possible.

Problem:

(a) Draw a probability tree (see Morgan article in Glickman and Gough, page 24-25), and evaluate the probability, $P[B]$, that the search will point to brake failure as the cause of the crash.

(b) Use Bayes' theorem to evaluate the "posterior probability" that the brakes failed

(i) given that the investigation indicates this cause

(ii) given that the investigation fails to indicate this cause

You are welcome to use equations, tables, or any other method that makes sense to you.

(c) Is your subjective judgment about the posterior probabilities mentioned in Part (b) consistent with the numerical values obtained using Bayes' theorem? In other words, does anything in part (b) seem counterintuitive? Why or why not?