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Math 181
Recitation Quiz 5 (3.1, 3.2 and 3.4)
SOLUTIONS

Please notify me if you feel there is a typo in need of correction.

1. Find the domain for the function $f(x)=\sqrt{1-2^{-t}}$

   SOLN: $1-2^{-t}\ge 0 \Rightarrow 1\ge 2^{-t} \Rightarrow \ln(1)\ge \ln(2^{-t}) \Rightarrow 0\ge(-t)\ln(2) \Rightarrow 0\ge(-t)\Rightarrow t\ge 0$

2. Find $\lim_{x\to 2^-} e^{3/(2-x)}$

   SOLN: $\lim_{x\to 2^-}\frac{3}{2-x}=+\infty \Rightarrow \lim_{x\to 2^-} e^{3/(2-x)}=\lim_{t\to+\infty} e^t=\infty$

3. (15 points) A bacteria culture grows with constant relative growth rate. After 2 hours there are 600 bacteria and after 8 hours the count is 89,000. Find the initial population and the expression for the population *p* after *t* hours. When will the population reach 100,000? (Round all values to 2 decimal places)

   SOLN: $y=kb^t$.

   $y(2)=600 \Rightarrow 600=kb^2$

   $y(8)=89000 \Rightarrow 89000=kb^8$

   $\frac{kb^8}{kb^2}=\frac{89000}{600}\approx 148.33 \Rightarrow b=(148.33)^{1/6}\approx 2.30$

   $kb^2=600 \Rightarrow k=\frac{600}{b^2}\approx\frac{600}{2.3^2}=113.42$

   $y=(113.42)(2.30)^t$

   $$100000=(113.42)(2.30)^t \Rightarrow \ln\frac{100000}{113.42}=\ln(2.30)^t \Rightarrow \frac{\ln\frac{100000}{113.42}}{\ln(2.30)}=t \Rightarrow t\approx 8.14hr$$

4. (15 pints) The half life of a compound is 28 years. Suppose we have an initial quantity of 1000g. Find the mass that remains after *t* years. When will there be less than 1g left? (Round all values to 3 places)

   SOLN: $y=kb^t$

   $\frac{k}{2}=kb^{28} \Rightarrow \frac{1}{2}=b^{28} \Rightarrow b=\left(\frac{1}{2}\right)^{1/28}\approx 0.976$

   $y=1000(0.976)^t$

   $$1=1000(0.976)^T \Rightarrow \ln\frac{1}{1000}=\ln(0.976)^T \Rightarrow T=\frac{\ln\frac{1}{1000}}{\ln(0.976)}\approx 284.356$$

5. In the theory of relativity, the mass of a particle with speed v is given by $m = f(v) = \dfrac{m_0}{\sqrt{1-v^2/c^2}}$ where m_0 is the rest mass of the particle and c is the speed of light in a vacuum. Find the inverse function of f and explain its meaning

SOLN:

$$v = \frac{m_0}{\sqrt{1-f^2/c^2}} \;\Rightarrow\; \frac{v}{m_0} = \frac{1}{\sqrt{1-f^2/c^2}} \;\Rightarrow\; \left(\frac{m_0}{v}\right)^2 = 1 - \frac{f^2}{c^2}$$

$$\Rightarrow\; -\left(\frac{m_0}{v}\right)^2 + 1 = \frac{f^2}{c^2} \;\Rightarrow\; c^2\left(-\left(\frac{m_0}{v}\right)^2 + 1\right) = f^2 \;\Rightarrow\; f = \pm\sqrt{c^2\left(1-\left(\frac{m_0}{v}\right)^2\right)}$$

$$\therefore\; f^{-1}(v) = \sqrt{c^2\left(1-\left(\frac{m_0}{v}\right)^2\right)}$$

The original function is the mass as a function of speed.
The inverse would be the speed given mass.
Note that speed is positive, which is why we only choose the + and not the –.

6. Find a formula for the inverse of the function $y = \dfrac{1+e^x}{1-e^x}$. What is the domain of the original function? What is the domain of the inverse function?

SOLN:

$$x = \frac{1+e^y}{1-e^y} \;\Rightarrow\; x(1-e^y) = 1+e^y \;\Rightarrow\; -xe^y - e^y = 1-x$$

$$\Rightarrow\; e^y = -\frac{1-x}{x+1} \;\Rightarrow\; y = \ln\left(\frac{x-1}{x+1}\right)$$

$$y^{-1}(x) = \ln\left(\frac{x-1}{x+1}\right)$$

The domain of the original function is all x except $1 = e^x$ $(x = 0)$.

The domain of the inverse function is $\dfrac{x-1}{x+1} > 0 \;\Rightarrow\; (-\infty,-1)\cup(1,\infty)$.

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7. (10 points each) Solve the following equations for x. Round your answer to 2 decimal places and be sure to check your solution.

a) $e^{2x+3} - 7 = 0$

SOLN: $e^{2x+3} = 7 \Rightarrow 2x + 3 = \ln 7 \Rightarrow x = \dfrac{-3 + \ln 7}{2} \approx -0.53$

check: $e^{2(-.53)+3} \approx 6.96$

b) $\ln(5 - 2x) = -3$

SOLN: $e^{-3} = 5 - 2x \Rightarrow x = \dfrac{e^{-3} - 5}{-2} \Rightarrow x = \dfrac{5 - e^{-3}}{2} \approx 2.48$

check: $\ln(5 - 2(2.48)) \approx -3.22$

8. Find dy/dx for the equation $xe^y + ye^x = 1$

SOLN: $xe^y y' + e^y + ye^x + e^x y' = 0 \Rightarrow y'(xe^y + e^x) = -e^y - ye^x \Rightarrow y' = \dfrac{-e^y - ye^x}{xe^y + e^x} = -\dfrac{ye^x + e^y}{xe^y + e^x}$